**Refresh Representing vectors using components**

To express a vector in terms of components, you first introduce a system of coordinate axes with which to work, as illustrated in Figure 1. You can choose any perpendicular directions for the axes, but usually you choose them to be horizontal and vertical, or you align them with directions intrinsic to the situation that you’re dealing with. You label the axes $x$ and $y$ (in two dimensions), or $x$, $y$ and $z$ (in three dimensions), in the usual way.

![Figure 1](image1.png)

**Figure 1** A vector in a coordinate system (a) in two dimensions (b) in three dimensions

We denote the vectors of magnitude 1 in the directions of the $x$-, $y$- and $z$-axes by $i$, $j$ and $k$, respectively, as shown in Figure 2. These vectors are called the **Cartesian unit vectors**. In general, a **unit vector** is a vector with magnitude 1.

![Figure 2](image2.png)

**Figure 2** The Cartesian unit vectors (a) in two dimensions (b) in three dimensions

Every vector can be expressed as the sum of scalar multiples of the Cartesian unit vectors, as illustrated in Figure 3. For example, in Figure 3(a), $\mathbf{u} = 3\mathbf{i} + (-2\mathbf{j}) = 3\mathbf{i} - 2\mathbf{j}$, and in Figure 3(b), $\mathbf{v} = \frac{3}{2}\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. 
Figure 3  Vectors expressed as sums of scalar multiples of the Cartesian unit vectors

We make the following definitions, illustrated in Figure 4.

**Component form of a vector**

If \( \mathbf{v} = a \mathbf{i} + b \mathbf{j} \), then the expression \( a \mathbf{i} + b \mathbf{j} \) is called the component form of \( \mathbf{v} \). The scalars \( a \) and \( b \) are called the \textbf{i-component} and \textbf{j-component}, respectively, of \( \mathbf{v} \).

Similarly, if \( \mathbf{v} = a \mathbf{i} + b \mathbf{j} + c \mathbf{k} \), then the expression \( a \mathbf{i} + b \mathbf{j} + c \mathbf{k} \) is called the component form of \( \mathbf{v} \). The scalars \( a \), \( b \) and \( c \) are called the \textbf{i-component}, \textbf{j-component} and \textbf{k-component}, respectively, of \( \mathbf{v} \).

Figure 4  General vectors expressed as sums of scalar multiples of the Cartesian unit vectors

For example, the component form of the vector \( \mathbf{u} \) in Figure 3 on the previous page is \( \mathbf{u} = 3 \mathbf{i} - 2 \mathbf{j} \) and its i- and j-components are 3 and −2, respectively.

The components of a vector represent the ‘amount of the vector’ in the positive direction of each axis.

The i- and j-components of a two-dimensional vector are alternatively called the x-component and y-component, respectively. Similarly, the i-, j- and k-components of a three-dimensional vector are alternatively called the x-component, y-component and z-component, respectively.
**Activity 1  Expressing vectors in component form**

Express each of the vectors in the diagram below in component form.

![Diagram showing vectors p, q, and r]

**Solution**
The component forms are $\mathbf{p} = 0i + 3j = 3j$, $\mathbf{q} = 3i + 4j$ and $\mathbf{r} = 2i - 3j$.

There is a useful alternative notation for expressing a vector in component form as follows.

**Alternative component form of a vector**
The vector $ai + bj$ can be written as $\begin{pmatrix} a \\ b \end{pmatrix}$.

The vector $ai + bj + ck$ can be written as $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

A vector written in this form is called a **column vector**.

For example,

$3i - 2j = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

The first number in a column vector is its $i$-component, the second number is its $j$-component, and, in three dimensions, the third number is its $k$-component.

**Activity 2  Writing vectors as columns**

Write the vectors in Activity 1 as column vectors.

**Solution**
The column forms of these vectors are $\mathbf{p} = 0i + 3j = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$, $\mathbf{q} = 3i + 4j = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\mathbf{r} = 2i - 3j = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$.
In general, if you are asked to write a vector in component form, then you can use either of the two types of component form. Both forms are used in this module. Sometimes one form is more convenient than the other.

It is useful to remember that, in two dimensions,
\[ \mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \]

and, in three dimensions,
\[ \mathbf{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \]

The zero vector, like any other vector, can be expressed in component form. In two dimensions,
\[ \mathbf{0} = 0\mathbf{i} + 0\mathbf{j} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \]

and, in three dimensions,
\[ \mathbf{0} = 0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \]