**Refresh Trigonometric identities**

MST224 expects you to be familiar with a number of trigonometric identities (MST224 HB p. 14) and they are listed here – the associated problem sheets will give you some practice in using some of these expressions without a calculator.

**Pythagoras’s theorem** states that for any right-angled triangle, if \( c \) is the length of the hypotenuse (the side opposite the right angle) and \( a \) and \( b \) are the lengths of the other two sides, then
\[
c^2 = a^2 + b^2.
\]

This leads to the following trigonometric identities:
\[
\sin^2 \theta + \cos^2 \theta = 1,
\]
\[
\tan^2 \theta + 1 = \sec^2 \theta,
\]
\[
1 + \cot^2 \theta = \cosec^2 \theta.
\]

**Addition formulas**
\[
\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta,
\]
\[
\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta,
\]
\[
\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta,
\]
\[
\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta,
\]
\[
\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta},
\]
\[
\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}.
\]
\[
\sin \alpha \cos \beta = \frac{1}{2} \sin(\alpha + \beta) + \frac{1}{2} \sin(\alpha - \beta),
\]
\[
\cos \alpha \sin \beta = \frac{1}{2} \sin(\alpha + \beta) - \frac{1}{2} \sin(\alpha - \beta),
\]
\[
\cos \alpha \cos \beta = \frac{1}{2} \cos(\alpha + \beta) + \frac{1}{2} \cos(\alpha - \beta),
\]
\[
\sin \alpha \sin \beta = \frac{1}{2} \cos(\alpha - \beta) - \frac{1}{2} \cos(\alpha + \beta).
\]

In particular, these formulas give
\[
\sin(\alpha + 2\pi) = \sin \alpha, \quad \cos(\alpha + 2\pi) = \cos \alpha, \quad \tan(\alpha + \pi) = \tan \alpha;
\]
\[
\sin(-\alpha) = -\sin \alpha, \quad \cos(-\alpha) = \cos \alpha, \quad \tan(-\alpha) = -\tan \alpha.
\]

**Double-angle formulas**
\[
\sin 2\alpha = 2 \sin \alpha \cos \alpha,
\]
\[
\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 1 - 2 \sin^2 \alpha = 2 \cos^2 \alpha - 1,
\]
\[
\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha},
\]
\[
\sin^2 \alpha = \frac{1}{2}(1 - \cos 2\alpha),
\]
\[
\cos^2 \alpha = \frac{1}{2}(1 + \cos 2\alpha).
\]

**Cosines of related angles**
\[
\cos(\frac{\pi}{2} - \alpha) = \sin \alpha, \quad \cos(\frac{\pi}{2} + \alpha) = -\sin \alpha,
\]
\[
\cos(\pi - \alpha) = -\cos \alpha, \quad \cos(\pi + \alpha) = -\cos \alpha.
\]