Equations that contain trigonometric functions of the unknown(s) are called **trigonometric equations**. Simple trigonometric equations such as

\[
\sin \theta = \frac{1}{2}, \quad \cos \theta = 0.3 \quad \text{and} \quad \tan \theta = -1
\]

occur frequently when you’re working with trigonometric functions.

One solution of such a simple trigonometric equation can be obtained by using an inverse trigonometric function. For instance, one solution of the equation \( \sin \theta = \frac{1}{2} \) is

\[
\theta = \sin^{-1} \left( \frac{1}{2} \right) = \frac{\pi}{6}.
\]

However, there are other solutions such as \( \frac{5\pi}{6} \) and \( -\frac{7\pi}{6} \), as illustrated in the figure below. In fact there are infinitely many solutions of the equation \( \sin \theta = \frac{1}{2} \), since the sine function is periodic.

The graph of the sine function, with solutions of \( \sin x = \frac{1}{2} \).

Usually when you want to solve a simple trigonometric equation, it’s best to begin by finding all the solutions that lie in an interval of length \( 2\pi \) (360°), such as \([0, 2\pi]\) or \([−\pi, \pi]\). There are usually two such solutions. Then you can obtain any other solutions that you want by adding integer multiples of \( 2\pi \) to the first solutions that you found.

The two sections below illustrate two methods for finding all the solutions of a simple trigonometric equation that lie in a particular interval of length \( 2\pi \).
To solve an equation of the form \( \sin \theta = c, \cos \theta = c \) or \( \tan \theta = c \) by using the ASTC diagram

(This method does not apply if \( c = 0 \), or if the equation is \( \sin \theta = c \) or \( \cos \theta = c \) where \( c = \pm 1 \).)

1. Use the ASTC diagram to find the quadrants of the solutions (there will be two such quadrants).
2. For each of these quadrants, draw a sketch showing the line \( OP \) in that quadrant. On each sketch, mark the associated angle \( \theta \) that lies in the interval \([0, 2\pi]\), and the acute angle \( \phi \) between \( OP \) and the x-axis. (You can use the interval \([-\pi, \pi]\) instead of \([0, 2\pi]\), but you should use the same interval for each sketch.)

3. Find \( \phi \) by applying the appropriate inverse trigonometric function to the equation \( \sin \phi = |c|, \cos \phi = |c| \) or \( \tan \phi = |c| \), as appropriate.
4. Use your sketches to find two values of \( \theta \) in the interval \([0, 2\pi]\) (or in the interval \([-\pi, \pi]\)) that are solutions of the equation.
5. If required, add multiples of \( 2\pi \) to obtain further solutions, or solutions in a different interval.

To solve an equation of the form \( \sin \theta = c, \cos \theta = c \) or \( \tan \theta = c \) by using a sketch graph

1. Sketch a graph of the relevant trigonometric function on the interval \([-\pi, \pi]\).
2. Find one solution of the equation by using the appropriate inverse trigonometric function, and mark it on your sketch.
3. Use the symmetry of the graph to find any other solutions in the interval \([-\pi, \pi]\) (usually there is one further such solution).
4. If required, add multiples of \( 2\pi \) to obtain further solutions, or solutions in a different interval.