**Refresh Solving linear equations**

A **linear** equation is one in which, after you’ve expanded any brackets and cleared any fractions, each term is either a constant term or a constant value times a variable. A linear equation in one unknown contains just one unknown.

For example,

\[ 2x - 5 = 8x \]

is a linear equation in the single unknown \( x \). Usually, a linear equation in one unknown has exactly one solution.

**Strategy:**

To solve a linear equation in one unknown

Use the rules for rearranging equations to obtain successive equivalent equations. Aim to obtain an equation in which the unknown is alone on one side, with only a number on the other side. To achieve that, do the following, in order.

1. Clear any fractions and multiply out any brackets. To clear fractions, multiply through by a suitable expression.
2. Add or subtract terms on both sides to get all the terms in the unknown on one side, and all the other terms on the other side. Collect like terms.
3. Divide both sides by the coefficient of the unknown.
Example 1  Solving linear equations

Solve the following equations.

(a) \( \frac{x}{5} - 4 = 3(4 - x) \)  \hspace{1cm} (b) \( \frac{1}{a} - 1 = \frac{1}{7a} \)

Solution

(a) \( \frac{x}{5} - 4 = 3(4 - x) \)

\( \text{There is a fraction with denominator 5, so multiply through} \text{ by 5 to clear it.} \)

\( x - 20 = 15(4 - x) \)

\( \text{Multiply out the brackets.} \)

\( x - 20 = 60 - 15x \)

\( \text{Get all the terms in the unknown on one side, and all the} \text{ other terms on the other side. Collect like terms.} \)

\( x + 15x = 60 + 20 \)

\( 16x = 80 \)

\( \text{Divide both sides by 16, the coefficient of the unknown.} \)

\( x = 5 \)

The solution is \( x = 5 \).

\( \text{If you wish, check the answer by substituting into the original} \text{ equation, as follows.} \)
Check: if \( x = 5 \),
\[
\text{LHS} = \frac{5}{9} - 4 = 1 - 4 = -3,
\]
and
\[
\text{RHS} = 3(4 - 5) = 3 \times (-1) = -3.
\]
Since \( \text{LHS} = \text{RHS} \), \( x = 5 \) satisfies the equation.

(b) \[
\frac{1}{a} - 1 = \frac{1}{7a}
\]
To clear the fractions, multiply through by a common multiple of the denominators, such as the lowest common multiple, \( 7a \). For this to be guaranteed to give an equivalent equation, you have to assume that \( 7a \neq 0 \), that is, \( a \neq 0 \).

Assume that \( a \neq 0 \).
\[
\frac{7a}{a} - 7a = \frac{7a}{7a}
\]
Simplify, then proceed as in part (a).
\[
7 - 7a = 1
\]
\[
7 - 1 = 7a
\]
\[
6 = 7a
\]
\[
\frac{6}{7} = a
\]
The value \( a = \frac{6}{7} \) satisfies the assumption \( a \neq 0 \), so it is the solution.