**Refresh Rearranging equations**

When you **rearrange** an equation the unknown variable you are trying to find is placed by itself on one side of the equals sign and all the other variables are on the other side. When you rearrange an equation, the original equation and the new one are said to be **equivalent**, or different **forms** of the same equation. Rewriting an equation as a simpler equation is called **simplifying** the equation.

### Rearranging equations

Carrying out any of the following operations on an equation gives an equivalent equation.
- Rearrange the expressions on one or both sides.
- Swap the sides.
- Do the same thing to both sides.

### Doing the same thing to both sides of an equation

Doing any of the following things to both sides of an equation gives an equivalent equation.
- Add something.
- Subtract something.
- Multiply by something (provided that it is non-zero).
- Divide by something (provided that it is non-zero).
- Raise to a power (provided that the power is non-zero, and that the expressions on each side of the equation can take only non-negative values).
To make a variable the subject of an equation

(This works for some equations but not all.)

Use the rules for rearranging equations to obtain successive equivalent equations. Aim to obtain an equation in which the required subject is alone on one side. To achieve this, do the following, in order.

1. Clear any fractions and multiply out any brackets. To clear fractions, multiply through by a suitable expression.
2. Add or subtract terms on both sides to get all the terms containing the required subject on one side, and all the other terms on the other side. Collect like terms.
3. If more than one term contains the required subject, then take it out as a common factor.
4. Divide both sides by the expression that multiplies the required subject.

Change the side, change the sign

Suppose you want to make \( y \) the subject of the equation shown below

\[
2x + 7 = z - y
\]

You can achieve this by first adding the term \( y \) to each side.

\[
2x + 7 + y = z - y + y
\]

Then subtracting \( 2x \) and subtracting 7 from both sides

\[
2x + 7 + y - 2x - 7 = z - 2x - 7
\]

\[
y = \frac{z - 2x - 7}{-1}
\]

Compare this rearranged equation to the original equation to see that when we move a term from one side to the other we change its sign. ‘Change the side, change the sign’.

- Multiplying each term on both sides of an equation by something (provided that it is non-zero) gives an equivalent equation.
- Dividing each term on both sides of an equation by something (provided that it is non-zero) gives an equivalent equation.
Cross-multiplying

Suppose that you want to clear the fractions in the equation
\[ \frac{x}{2} = \frac{x + 1}{3}. \]
You can do this by multiplying through by 2, and then multiplying through by 3. This gives the equation
\[ 3x = 2(x + 1). \]
You can see that the overall effect on the original equation is that you have ‘multiplied diagonally across the equals sign’, like this:
\[ \frac{x}{2} \times \frac{x + 1}{3} \text{ gives } 3x = 2(x + 1). \]

This technique is called **cross-multiplying**. You can use it as a shortcut for multiplying through, whenever you have an equation of the form fraction = fraction. The general rule is summarised in the box below.

If only one side of an equation is a fraction, then you can still cross-multiply (the other side can be thought of as a fraction with denominator 1).

For example,
\[ \frac{x}{2} = x + 1 \text{ is equivalent to } x = 2(x + 1). \]