**Refresh Multiplying out brackets**

Also called expanding the brackets or removing the brackets.

Multiplying out the brackets below to remove them

\[ 4c(2 + 3a). \]

**Rule:** Multiply each term inside the brackets by the **multiplier**. The multiplier is the expression **outside** the bracket. In this example the multiplier is just 4c.

\[ 4c(2 + 3a) = 4c \times 2 + 4c \times 3a = 8c + 12ac. \]

When you multiply out the brackets, it doesn’t matter whether the multiplier is before or after the brackets.

For example,

\[(15k - 1)m = 15km - m.\]

Also

\[m(15k - 1) = 15km - m.\]

These are obviously the same.

Removing brackets with a minus sign in front just requires each term inside the bracket to be multiplied by the minus sign.

For example,

\[-(a + 25b - s) = -a - 25b + s.\]

A quick way to multiply by a minus sign is just to change the sign of each term inside the brackets.

Remember that for any expressions \(A\) and \(B\), the negative of \(A - B\) is \(B - A\). It’s helpful to remember this fact. For example, it tells you immediately that

\[-(x - y) = y - x\]

and

\[-(n - 3n^2) = 3n^2 - n.\]

When removing brackets with a plus sign in front you just keep the original sign for each term.

For example,

\[+(k - y) = k - y.\]
To multiply out brackets in an expression with more than one term

\[ x + 3x(1 + 3x) \]

1. Identify the terms. Each term after the first starts with a plus or minus sign that isn’t inside brackets.
2. Multiply out the brackets in each term. Include the sign (plus or minus) at the start of each resulting term.
3. Collect any like terms.

For example,

\[ x + 3x(1 + 3x) = x + 3x + 9x^2 \]

now collect like terms on the right-hand side to get \( 4x + 9x^2 \).

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**Example 1**  *Multiplying out brackets when there’s more than one term*

Multiply out the brackets in the expression

\[ 2c(c + d) + 5c^2 - d(c - d) \]

and simplify your answer.

**Solution**

Identify the terms; you might find it helpful to mark them. Multiply out the brackets in each term individually to obtain a new expression with five terms. Finally, collect any like terms.

\[
2c(c + d) + 5c^2 - d(c - d) = 2c^2 + 2cd + 5c^2 - cd + d^2 = 7c^2 + cd + d^2
\]

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**Multiplying out two pairs of brackets**

\[ (x + 5)(7 - x) \]

It doesn’t matter which of the two bracketed expressions you choose to be the multiplier but it’s easier to use the second one namely \( (7 - x) \).

Multiply each term in the first bracket by the multiplier \( (7 - x) \).

\[
(x + 5)(7 - x) = x(7 - x) + 5(7 - x)
\]

Multiply out the brackets in each term

\[
x(7 - x) + 5(7 - x) = 7x - x^2 + 35 - 5x
\]

Then collect any like terms on the right-hand side to get \(-x^2 + 2x + 35\).

The convention is to write the powers of \( x \) in descending order.

This approach will work for any brackets with any number of terms.

If you have two brackets to multiply and each bracket has two terms you can use FOIL to help.
Example 2 Using FOIL to multiply out brackets

Multiply out the brackets in the expression

\((x + 2)(3x - 5)\),

and simplify your answer.

Solution

First: \(x \times 3x = 3x^2\).

Outer: \(x \times (-5) = -5x\).

Inner: \((+2) \times 3x = +6x\).

Last: \((+2) \times (-5) = -10\).

\((x + 2)(3x - 5) = 3x^2 - 5x + 6x - 10 = 3x^2 + x - 10\)

Special circumstances

Try to remember these important results:

**Difference of two squares**

For any expressions \(A\) and \(B\)

\((A + B)(A - B) = A^2 - B^2\)

**Squaring brackets**

For any expressions \(A\) and \(B\),

\((A + B)^2 = A^2 + 2AB + B^2\)

and

\((A - B)^2 = A^2 - 2AB + B^2\)

Example 3 Multiplying out squared brackets efficiently

Multiply out the brackets in the following expressions, and simplify your answers.

(a) \((x + 3y)^2\)  
(b) \((4h - 1)^2\)

Solution

(a) The answer is the square of \(x\), plus twice the product of \(x\) and \(3y\), plus the square of \(3y\).

\((x + 3y)^2 = x^2 + 2 \times x \times (3y) + (3y)^2 = x^2 + 6xy + 9y^2\)

(b) The answer is the square of \(4h\), minus twice the product of \(4h\) and \(1\), plus the square of \(1\).

\((4h - 1)^2 = (4h)^2 - 2 \times (4h) \times 1 + 1^2 = 16h^2 - 8h + 1\)