Finding factors and multiples

If an algebraic expression can be written in the form

\[ \text{something} \times \text{another something} \]

then both ‘somethings’ are factors of the expression, and the expression is a multiple of both ‘somethings’.

For example, the equation

\[ a^2b = a \times ab \]

shows a and ab are factors of \( a^2b \), and that \( a^2b \) is a multiple of both a and ab.

Two or more algebraic expressions may also have common factors and common multiples.

A common factor of two or more algebraic expressions is an expression that is a factor of all of them.

For example, the expression \( a \) is a common factor of the two expressions \( a^2b \) and \( abc \) because

\[ a^2b = a \times ab \text{ and } abc = a \times bc. \]

A common multiple of two or more algebraic expressions is an expression that is a multiple of all of them. For example, the expression \( abcd \) is a common multiple of the two expressions

\[ ab \text{ and } bc, \]

because

\[ abcd = ab \times cd \text{ and } abcd = bc \times ad. \]

The highest common factor means a common factor that is a multiple of all other common factors. Similarly, lowest common multiple means a common multiple that is a factor of all other common multiples.
Example 1  Finding HCFs and LCMs of algebraic expressions
Consider the expressions
\[ 10a^6 \text{ and } 15a^8b^3. \]

(a) Find the highest common factor of the expressions, and write each expression in the form

\[ \text{highest common factor} \times \text{something}. \]

(b) Find the lowest common multiple of the expressions, and, for each expression, write the lowest common multiple in the form

\[ \text{the expression} \times \text{something}. \]

Solution

(a)

First consider the coefficients. The largest integer that is a factor of both 10 and 15 (that is, their HCF) is 5.

Then consider the powers of \(a\). The largest power of \(a\) that is a factor of both \(a^6\) and \(a^8\) is \(a^6\). (Note that \(a^6\) is a factor of both \(a^6\) and \(a^8\) because \(a^6 = a^6 \times 1\) and \(a^8 = a^6 \times a^2\).)

Finally, consider the powers of \(b\). There is no power of \(b\) in \(10a^6\), so there is no power of \(b\) in the highest common factor.

The highest common factor of the two terms is
\[ 5a^6. \]

The expressions can be written as
\[ 10a^6 = 5a^6 \times 2 \quad \text{and} \quad 15a^8b^3 = 5a^6 \times 3a^2b^3. \]

(b)

First consider the coefficients. The smallest positive integer that is a multiple of both 10 and 15 (that is, their LCM) is 30.

Then consider the powers of \(a\). The smallest power of \(a\) that is a multiple of both \(a^6\) and \(a^8\) is \(a^8\).

Finally, consider the powers of \(b\). The smallest power of \(b\) that is a multiple of both ‘no power of \(b\)’ and \(b^3\) is \(b^3\).

The lowest common multiple of the two terms is
\[ 30a^8b^3. \]

It can be written as
\[ 30a^8b^3 = 10a^6 \times 3a^2b^3 \quad \text{and} \quad 30a^8b^3 = 15a^8b^3 \times 2. \]
Factorising an expression means writing it as the product of two or more expressions, neither of which is 1 (and, usually, neither of which is \(-1\)).

If all of the terms of an expression have a common factor other than 1, then the expression can be factorised. For example, consider the expression

\[x^3y + xy\]

The terms of this expression, \(x^3y\) and \(xy\), have \(xy\) as a common factor. So the expression can be written as

\[xy \times x^2 + xy \times 1\]

From your work on multiplying out brackets, you know that this is the same as

\[xy(x^2 + 1)\]

The original expression has now been factorised. We say that we have taken out the common factor \(xy\).

**Strategy:**

To take out a common factor from an expression

1. Find a common factor of the terms (usually the highest common factor).
2. Write the common factor in front of a pair of brackets.
3. Write what’s left of each term inside the brackets.

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**Example 2  Taking out a common factor**

Factorise the expression \(-8g^5 + 4g^2h^2 - 2g^2\).

**Solution**

The highest common factor of the three terms is \(2g^2\).

\[-8g^5 + 4g^2h^2 - 2g^2 = 2g^2(-4g^3 + 2h^2 - 1)\]