Practice  Essential basics - set 1

Exercise 1.1
Evaluate the following expressions without using a calculator then use your calculator to check your answers.

(a) $2 - (122 - 10 \times 3) + 17$  
(b) $12 + (5 + 4 \times 3^2)$  
(c) $19 - (15 - 7 \times 2^2 \times 3^2)$  
(d) $(11 \times 4^2) - (19 - 2 \times 6^2)$

Exercise 1.2
Round the following numbers to two significant figures.

(a) 4.235  
(b) 0.09776  
(c) 102  
(d) 17.49

Exercise 1.3
Simplify the following expressions.

(a) $12m + 15m - 26m$  
(b) $0.5XY^2 + 0.1XY^2$  
(c) $5p - p$  
(d) $\frac{1}{3}d - 2d$

Exercise 1.4
Evaluate the expression

$4x^2 - 5y$

when $x = 2$ and $y = -3$.

Exercise 1.5
Multiply out the brackets in the following expressions, and simplify as far as possible.

(a) $(a + 7)(a - 1)$  
(b) $(c + 4)(4c - 3)$  
(c) $(3x - 4y)(3x + 4y)$  
(d) $(2A + B)^2$  
(e) $(9 - 2t)(5t - 1)$  
(f) $(3a - \frac{1}{2})^2$  
(g) $\left(x + \frac{1}{x}\right)^2$  
(h) $2(x + 3)(x - 4) - (x - 6)^2$  
(i) $(\sqrt{x} + 2\sqrt{y})(\sqrt{x} - 3\sqrt{y})$  
(j) $(a + 2b)(a - 2b + c)$  
(k) $(a^2 + 3a - 1)(a^3 - a + 2)$

Exercise 1.6
Show that:

(a) $3g$ is the highest common factor of $12gk$ and $3gw$.
(b) $y^3$ is the highest common factor of $x^2y^3$ and $h^5y^3$.
(c) $5k$ is the highest common factor of $15kz$, $10kx^2$ and $200ky^3$.

Exercise 1.7
Simplify the following expressions.

(a) $2a^3 - 3a - 2a^3 - 3a$  
(b) $2m + n - 5m + 2n + 3m$  
(c) $b + 2b + 3b - 6b$
**Exercise 1.8**

Write the following terms in their shortest forms.

(a) \( y \times z \times 6 \times x \times 4 \)  
(b) \( 7p \times 2qr \)  
(c) \( QR \times G \times 5F \)  
(d) \( 2 \times a \times a \times 3 \times a \)  
(e) \( m \times n \times m \times 4 \)  
(f) \( 5y \times 2yx \)  
(g) \( 4AB \times 4AB \)

**Exercise 1.9**

Write the following terms in their shortest forms.

(a) \( 4q \times (−2p) \)  
(b) \( -B^3 \times (−5B) \)  
(c) \( -a \times (−b) \times (−a) \)

**Exercise 1.10**

Write the following terms in their shortest forms.

(a) \( +(-ab) \)  
(b) \( -(−6x^2) \)  
(c) \( -(2M^4) \)  
(d) \( +(−7y) \)  
(e) \( +(5p) \)  
(f) \( -(−\frac{3}{4}n) \)

**Exercise 1.11**

Mark the terms in the following expressions.

(a) \( -2a - (−5a^2) + (−4a) \)  
(b) \( 2x \times 4xy - 2y \times (−5x) \)

**Exercise 1.12**

Simplify the following expressions.

(a) \( -2a - (−5a^2) + (−4a) \)  
(b) \( 2x \times 4xy - 2y \times (−5x) \)

**Exercise 1.13**

Show that \( 3xy \) is a factor of \( 3xy^3 \), by writing \( 3xy^3 \) in the form  
\[ 3xy \times \text{something} \]

**Exercise 1.14**

(a) Show that \( z \) is a common factor of \( 2z \) and \( z^2 \).
(b) Show that \( p^2 \) is a common factor of \( p^2q^2 \) and \( p^2 \).
(c) Show that \( 2AB \) is a common factor of \( 2A^2B^2 \), \( 4A^2B \) and \( 8AB \).

**Exercise 1.15**

Find the highest common factor of the terms  
\[ 6ab^2c^2 \quad \text{and} \quad 9a^2b^5 \],

and write each term in the form  
\[ \text{highest common factor} \times \text{something} \]

**Exercise 1.16**

In each of the following parts, find the highest common factor of the terms and write each term in the form  
\[ \text{highest common factor} \times \text{something} \]

(a) \( 2ab^2 \) and \( 4ab \)  
(b) \( 3xy \) and \( 6y \)  
(c) \( 4p^3 \), \( 9p^2 \) and \( 2p^5 \)  
(d) \( 10r \) and \( 15s \)
Exercise 1.17
Factorise the following expressions.
(a) $ab + a^2$  
(b) $x^3y + yz$  
(c) $2w^2 + w^3$  
(d) $2z + 6z^4$

Exercise 1.18
Factorise the expression $3m^3 - 6m^2 + 3m^4$.

Exercise 1.19
Factorise the following expressions.
(a) $0.3m^2 - 0.6m + 0.9$  
(b) $\frac{1}{2}x - \frac{1}{2}x^2$
Solutions to Exercises

Solution 1.1

(a) \[2 - (122 - 10 \times 3) + 17 = 2 - (122 - 30) + 17 = 2 - (92) + 17 = -90 + 17 = -73\]

(b) \[12 + (5 + 4 \times 3^2) = 12 + (5 + 4 \times 9) = 12 + (5 + 36) = 12 + 41 = 53\]

(c) \[19 - (15 - 7 \times 2^2 \times 3^2) = 19 - (15 - 7 \times 4 \times 9) = 19 - (15 - 252) = 19 - (-237) = 19 + 237 = 256\]

(d) \[(11 \times 4^2) - (19 - 2 \times 6^2) = (11 \times 16) - (19 - 2 \times 36) = (176) - (19 - 72) = 176 - (-53) = 176 + 53 = 229\]

Solution 1.2

(a) \[4.235 = 4.2\] (to 2 s.f.)

(b) \[0.09776 = 0.098\] (to 2 s.f.)

(c) \[102 = 100\] (to 2 s.f.)

(d) \[17.49 = 17\] (to 2 s.f.)

Solution 1.3

(a) \[12m + 15m - 26m = (12 + 15 - 26)m = 1m = m\]

(b) \[0.5XY^2 + 0.1XY^2 = (0.5 + 0.1)XY^2 = 0.6XY^2\]

(c) \[5p - p = 5p - 1p = (5 - 1)p = 4p\]

(d) \[\frac{4}{3}d - 2d = (\frac{4}{3} - 2)d = (\frac{4}{3} - \frac{6}{3})d = -\frac{2}{3}d\]

Solution 1.4

If \(x = 2\) and \(y = -3\), then
\[4x^2 - 5y = 4 \times 2^2 - 5 \times (-3) = 4 \times 4 - (-15) = 16 + 15 = 31.\]

Solution 1.5

(a) \[(a + 7)(a - 1) = a^2 - a + 7a - 7 = a^2 + 6a - 7\]

(b) \[(c + 4)(4c - 3) = 4c^2 - 3c + 16c - 12 = 4c^2 + 13c - 12\]

(c) \[(3x - 4y)(3x + 4y) = 9x^2 + 12xy - 12xy - 16y^2 = 9x^2 - 16y^2\]

(d) \[(2A + B)^2 = (2A + B)(2A + B) = 4A^2 + 2AB + 2AB + B^2 = 4A^2 + 4AB + B^2\]

(e) \[(9 - 2t)(5t - 1) = 45t - 9 - 10t^2 + 2t = -10t^2 + 47t - 9\]

(f) \[(3a - \frac{2}{3})^2 = (3a - \frac{2}{3})(3a - \frac{2}{3}) = 9a^2 - 2a - 2a + \frac{4}{9} = 9a^2 - 4a + \frac{4}{9}\]

(g) \[\left(x + \frac{1}{x}\right)^2 = \left(x + \frac{1}{x}\right) \left(x + \frac{1}{x}\right) = x^2 + 1 + \left(\frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2}\]

(h) \[2(x + 3)(x - 4) - (x - 6)^2 = 2(x^2 - 4x + 3x - 12) - (x^2 - 12x + 36) = 2x^2 - 2x - 24 - x^2 + 12x - 36 = x^2 + 10x - 60\]

(i) \[(\sqrt{x} + 2\sqrt{y})(\sqrt{x} - 3\sqrt{y}) = x - 3\sqrt{xy} + 2\sqrt{xy} - 6y = x - \sqrt{3y} - 6y\]

(j) \[(a + 2b)(a - 2b + c) = a^2 - 2ab + ac + 2ab - 4b^2 + 2bc = a^2 - 4b^2 + ac + 2bc\]

(k) \[(a^2 + 3a - 1)(a^2 - a + 2) = a^5 + 3a^2 + 3a^4 - 3a^2 + 6a - a^3 + a - 2 = a^5 + 3a^4 - 2a^3 - a^2 + 7a - 2\]
Solution 1.6
(a) $12gk = 3g(4k)$ and $3gw = 3g(w)$ so $3g$ is the highest common factor.
(b) You can see that $y^3$ is the highest common factor just by inspection.
(c) $15kz = 5k(3z)$
$$10kx^2 = 5k(2x^2)$$
and $200gy^3 = 5k(40y^3)$
so $5k$ is the highest common factor.

Solution 1.7
(a) $2a^3 - 3a - 2a^3 - 3a = -6a$
(b) $2m + n - 5m + 2n + 3m = 3n$
(c) $b + 2b + 3b - 6b = 0$

Solution 1.8
(a) $y \times z \times 6 \times x \times 4 = 24xyz$
(b) $7p \times 2qr = 14pqr$
(c) $QR \times G \times 5F = 5FGQR$
(d) $2 \times a \times a \times 3 \times a = 6a^3$
(e) $m \times n \times m \times 4 = 4m^2n$
(f) $5y \times 2yx = 10xy^2$
(g) $4AB \times 4AB = 16A^2B^2$

Solution 1.9
Instead of ‘a positive times a negative gives a negative’, we sometimes say, informally, ‘a plus times a minus gives a minus’.
(a) A positive times a negative gives a negative.
$$4q \times (-2p) = -4q \times 2p$$
$$= -8pq$$
(b) A negative times a negative gives a positive.
$$-B^3 \times (-5B) = +B^3 \times 5B$$
$$= +5B^4$$
$$= 5B^4$$
(c) The first negative times the second negative gives a positive, then that positive times the third negative gives a negative.
$$-a \times (-b) \times (-a) = -a \times b \times a$$
$$= -a^2b$$

Remember that the overall sign is found in the same way as when you multiply several negative numbers together.

Solution 1.10
(a) $+(-ab) = -ab$
(b) $-(-6x^2) = +6x^2 = 6x^2$
(c) $-(2M^4) = -2M^4$
(d) $+(-7y) = -7y$
(e) $+(5p) = +5p = 5p$
(f) $-(\frac{3}{4}n) = -\frac{3}{4}n$

Solution 1.11
(a) Begin by marking the start of the first term.
$$-2a - (-5a^2) + (-4a)$$

Extend the line under the first term until you reach a plus or minus sign that isn’t inside brackets. That’s the start of the next term.
$$-2a - (-5a^2) + (-4a)$$

Extend the line under the second term until you reach a plus or minus sign that isn’t inside brackets. That’s the start of the next term.
$$-2a - (-5a^2) + (-4a)$$

Extend the line under the third term until you reach a plus or minus sign that isn’t inside brackets. This time you don’t reach one – you just reach the end of the expression. So this expression has three terms, as marked.
$$-2a - (-5a^2) + (-4a)$$

(b) Mark the start of the first term.
$$2x \times 4xy - 2y \times (-5x)$$

When you reach a plus or minus sign that isn’t inside brackets, that’s the start of the next term.
$$2x \times 4xy - 2y \times (-5x)$$

You don’t reach another plus or minus sign that isn’t inside brackets, so this expression has two terms.
$$2x \times 4xy - 2y \times (-5x)$$
Solution 1.12

(a) First identify the terms. Then simplify each term individually. Finally, collect like terms.

\[-2a - (-5a^2) + (-4a) = -2a + 5a^2 - 4a\]
\[= 5a^2 - 6a\]

(b) Identify the terms, then simplify each term individually. Finally, check for like terms – there are none here.

\[2x \times 4xy - 2y \times (-5x) = 8x^2y + 10xy\]

Solution 1.13

\[3xy^3 = 3xy \times y^2\]

Solution 1.14

(a) \(2z = z \times 2\) and \(z^2 = z \times z\).

So \(z\) is a common factor of the two terms.

(b) \(p^2q^3 = p^2 \times q^2\) and \(p^2 = p^2 \times 1\).

So \(p^2\) is a common factor of the two terms.

(c) \(2A^2B^2 = 2AB \times AB, 4A^2B = 2AB \times 2A\) and \(8AB = 2AB \times 4\).

So \(2AB\) is a common factor of the three terms.

Solution 1.15

First, consider the coefficients. The largest integer that divides both 6 and 9 exactly is 3. 3 is the highest common factor of 6 and 9.

Next, consider the powers of \(a\). The largest power of \(a\) that divides both \(a\) and \(a^2\) exactly is \(a\).

Then consider the powers of \(b\). The largest power of \(b\) that divides both \(b^7\) and \(b^5\) exactly is \(b^5\).

Finally, consider the powers of \(c\). The second term doesn’t contain \(c\) at all.

So, the highest common factor of the two terms is \(3ab^5\).

The terms can be written as

\[6ab^7c^2 = 3ab^5 \times 2b^2c^2\] and \(9a^2b^5 = 3ab^5 \times 3a\).

Solution 1.16

(a) The highest common factor of \(2ab^2\) and \(4ab\) is \(2ab\).

\(2ab^2 = 2ab \times b\) and \(4ab = 2ab \times 2\).

(b) The highest common factor of \(3xy\) and \(6y\) is \(3y\).

\(3xy = 3y \times x\) and \(6y = 3y \times 2\).

(c) The highest common factor of \(4p^3\), \(9p^2\) and \(2p^5\) is \(p^2\).

\(4p^3 = p^2 \times 4p, 9p^2 = p^2 \times 9\) and \(2p^5 = p^2 \times 2p^3\).

(d) The highest common factor of \(10r\) and \(15s\) is \(5\).

\(10r = 5 \times 2r\) and \(15s = 5 \times 3s\).

Solution 1.17

(a) \(ab + a^2 = a \times b + a \times a = a(b + a)\)

(b) \(x^3y + yz = y \times x^3 + y \times z = y(x^3 + z)\)

(c) \(2w^2 + w^3 = w^2 \times 2 + w^2 \times w = w^2(2 + w)\)

(d) \(2z + 6z^4 = 2z \times 1 + 2z \times 3z^3 = 2z(1 + 3z^3)\)

Solution 1.18

The highest common factor of the terms is \(3m^2\).

\(3m^3 - 6m^2 + 3m^4 = 3m^2 \times m - 3m^2 \times 2 + 3m^2 \times m^2\)
\[= 3m^2(m - 2 + m^2)\]
\[= 3m^2(m^2 + m - 2)\]
\[= 3m^2(m + 2)(m - 1)\]

Solution 1.19

(a) \(0.3m^2 - 0.6m + 0.9 = 0.3(m^2 - 2m + 3)\)

(b) \(\frac{1}{2}x - \frac{1}{3}x^2 = \frac{1}{6}x(1 - x)\)