Refresh Logarithmic applications

Logarithms are used in solving equations in which the unknown is an exponent (power).

Example 1
Solve each of the following exponential equations.

(a) \(5^x = 12\)
(b) \(25^{2x+1} = 15\)

Solution
(a) Since the quantities \(5^x\) and 12 are equal, so are their logarithms. Taking the common logarithm of each side of the equation gives
\[
\log 5^x = \log 12.
\]
Then, using Rule 3 of logarithms that \(\log a^n = n \log a\) (see Module 1), we obtain
\[
x \log 5 = \log 12
\]
so that
\[
x = \frac{\log 12}{\log 5} = 1.54 \text{ (to 2 d.p.).}
\]
Alternatively, using natural logarithms, we have
\[
\ln 5^x = \ln 12
\]
so that
\[
x \ln 5 = \ln 12,
\]
from which
\[
x = \frac{\ln 12}{\ln 5} = 1.54 \text{ (to 2 d.p.).}
\]

(b) Taking the natural logarithm of each side of \(25^{2x+1} = 15\) gives
\[
\ln 25^{2x+1} = \ln 15
\]
so that, by Rule 3 of logarithms,
\[
(2x + 1) \ln 25 = \ln 15,
\]
from which
\[
2x + 1 = \frac{\ln 15}{\ln 25}.
\]
Hence
\[
2x = \frac{\ln 15}{\ln 25} - 1
\]
so that
\[
x = \frac{1}{2} \left(\frac{\ln 15}{\ln 25} - 1\right) = -0.079 \text{ (to 3 d.p.).}
\]