**Refresh Integers**

**Multiples**

The multiples of a positive whole number are those positive whole numbers into which it divides exactly: for example, the multiples of 6 are 6, 12, 18, 24, . . . ; that is, the multiples of a number are that number multiplied by 1, 2, 3, 4, . . . . However, finding the common multiples of two or more numbers is more difficult, and to find the **lowest common multiple (LCM)** needs a knowledge of factors.

**Factors**

The process of finding all the whole numbers that divide exactly into a given whole number is called ‘finding the **factors**’. ‘Factorising’ means writing a number as the product of two or more whole numbers. For example, $48 = 12 \times 4$ and $48 = 8 \times 3 \times 2$.

A **prime number** is a positive whole number, other than 1, that is divisible only by itself and 1. Factorising a number so that each factor is a prime number is often very useful – this process is called ‘finding the prime factors’. For example, the factors of 48 which are prime are 2 and 3; the prime factors of 48 are

\[ 48 = 2 \times 2 \times 2 \times 2 \times 3 \quad (= 2^4 \times 3). \]

The procedure for finding the prime factors of a number $a$ are as follows. It is illustrated in Example 1(b) for $a = 60$.

**To find the prime factors of $a$**

1. Divide $a$ by 2 (the first prime number) repeatedly until the result is not divisible by 2. If the number of such divisions is $N$, then $2^N$ is a factor of $a$.
2. If the result is a prime number, the process is complete. If not, repeat the process for each successive prime number, replacing 2 by 3, 5, 7, . . . , as necessary.

**Example 1**

(a) Find the factors of 60.
(b) Find the prime factors of 60.
Solution

(a) The question is asking what pairs of whole numbers when multiplied together make 60. They are

\[ 1 \times 60, \quad 2 \times 30, \quad 3 \times 20, \quad 4 \times 15, \quad 5 \times 12, \quad 6 \times 10. \]

Thus the factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30 and 60.

(b) Divide 60 by \(2\): \(60 = 2 \times 30\). 30 is divisible by 2.
Divide 30 by \(2\): \(30 = 2 \times 15\). 15 is not divisible by 2.
Divide 15 by \(3\): \(15 = 3 \times 5\). 5 is prime.

\[
\begin{align*}
60 & \quad \text{divide by 2} \\
2 \times 30 & \quad \text{divide by 2} \\
2 \times 15 & \quad \text{divide by 3} \\
3 \times 5 & \quad \text{prime, so stop}
\end{align*}
\]

So the prime factors of 60 are 2, 2, 3, 5. Thus

\[ 60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5. \]

Highest common factor

The highest common factor (HCF) of any two positive whole numbers is the largest number which is a factor of both.

Or put another way, the HCF is the product of the lowest power of each prime factor common to both.

The following example illustrates how to find the HCF.

Example 2

What is the highest common factor of the numbers 18 and 30?

Solution

First, write each number in terms of its prime factors. We have

\[ 18 = 2 \times 3^2 \quad \text{and} \quad 30 = 2 \times 3 \times 5. \]

Taking the lowest power of the common prime factors gives

\[ \text{HCF of 18 and 30} = 2 \times 3 = 6. \]
Lowest common multiple

The **lowest common multiple (LCM)** of any two positive whole numbers is the smallest positive whole number which is a multiple of both.

Or put another way, the LCM is the product of the highest power of each prime factor occurring.

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**Example 3**

What is the lowest common multiple of each of the following sets of numbers?

(a) 10, 25.  
(b) 8, 24, 60.

**Solution**

(a) In terms of prime factors 10 can be written as $2 \times 5$ and 25 as $5 \times 5 = 5^2$. The LCM of 10 and 25 is the product of the highest power of each prime factor, that is

$$2 \times 5^2 = 2 \times 25 = 50.$$  
So the LCM of 10 and 25 is 50.

(b) $8 = 2^3$.

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$.

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$.

So the LCM of 8, 24, and 60 is $2^3 \times 3 \times 5 = 120$.  

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Again, this definition extends to three or more numbers.