Graphs of sine, cosine and tangent

The sine function \( f(x) = \sin x \)

In the above discussion, involving angles represented by \( \theta \) radians, \( \theta \) is any real number and \( \sin \theta \) is the number, between \(-1\) and \(1\), corresponding to \( \theta \). Thus we have the ingredients for a function. To emphasise this, we write

\[
f(x) = \sin x \quad (x \text{ is real})
\]

where we have used the customary \( x \) for input values. The graph of the function \( f(x) = \sin x \) is in (Figure 1).

![Figure 1](image1.png)

If this graph were extended in either direction more repetitions of the basic shape for \( x \) between 0 and \( 2\pi \) would be obtained.

Such a function is said to be **periodic**. The function \( f(x) = \sin x \) has **period** \( 2\pi \).

The cosine function \( f(x) = \cos x \)

The graph of the function \( f(x) = \cos x \) is shown in Figure 2.

![Figure 2](image2.png)

Like the sine function, the cosine function has domain the real numbers and is periodic, with period \( 2\pi \). Its values lie between the maximum value 1 and minimum value \(-1\), both of which occur infinitely many times.

The graph of the sine and cosine functions have the same shape: shifting the former to the left by \( \pi/2 \) gives the latter. This is summarised in the relationship

\[
\sin \left( x + \frac{\pi}{2} \right) = \cos x, \quad \text{for all } x.
\]
The tangent function

The graph of the function \( f(x) = \tan x \) is shown in Figure 3. It comprises infinitely many disjoint parts, each with the same shape.

![Graph of \( y = \tan x \)](image)

**Figure 3**

The tangent function is periodic but with period \( \pi \), but in other respects its graph looks very different from those for the sine and cosine functions. It has no maximum or minimum values.

Looking at values near to \( \pi/2 \), it can be seen that when \( x \) is less than \( \pi/2 \), but is getting closer and closer to \( \pi/2 \) from the left, the corresponding values of \( y \) get larger and larger; \( y \) is ‘approaching infinity’.

On the other hand when \( x \) is bigger than \( \pi/2 \) but getting closer and closer to \( \pi/2 \) from the right, the corresponding values of \( y \) get smaller and smaller; \( y \) is ‘approaching minus infinity’.

The function \( f(x) = \tan x \) is undefined at \( x = \pi/2 \), and so does not cross the line \( x = \pi/2 \). (Try finding \( \tan x = \pi/2 \) on your calculator.)

When a curve gets arbitrarily close to a particular line but never actually reaches or crosses it, the line is called an **asymptote** of that curve. In this case, the line \( x = \pi/2 \) is an asymptote of the graph of \( y = \tan x \).

The lines \( x = 3\pi/2, x = 5\pi/2, x = 7\pi/2, \ldots \) are also asymptotes of the graph of \( y = \tan x \), as are the lines \( x = -\pi/2, x = -3\pi/2, x = -5\pi/2, \) and so on.