**Refresh Exponential functions**

If $a$ is a *positive* real number and $n$ is a positive integer, then $a^n$ represents $a$ multiplied by itself $n$ times. A meaning can also be assigned in a natural way to $a^x$, where $x$ is any real number. When $x$ is not a positive integer, this is achieved as follows.

(i) $a^0 = 1$.

(ii) If $n$ is a positive integer, then $a^{-n} = 1/a^n$.

(iii) If $p, q$ are integers, with $q > 0$, then $a^{p/q} = (\sqrt[q]{a})^p$.

(iv) If $x$ is an irrational number, then the value of $a^x$ can be found to any desired accuracy by approximating $x$ more and more closely by rational numbers $p/q$, for which $a^{p/q}$ is defined by (iii) above.

With these definitions, we have the following rules for powers, where $x$ and $y$ are any real numbers:

$$a^{x+y} = a^x a^y \quad \text{and} \quad (a^x)^y = a^{xy}.$$  

For each positive real number $a$, the process just described for finding values of $a^x$ for any real number $x$ is a rule for the function

$$f(x) = a^x,$$

with domain $\mathbb{R}$. Each such function is called an **exponential function**, and the number $a$ is called the **base** of the function.