Refresh Area of a general triangle

It is known that the area of a triangle can be found using the formula
\[ A = \frac{1}{2}bh, \]
where \( b \) is the length of the base of the triangle and \( h \) is its height. However, the information known about a triangle does not always include its height. An alternative method of finding the area of a triangle can be used if two sides and the angle between them are known.

Suppose that in triangle \( XYZ \) in Figure 1 the lengths \( x \) and \( y \) and the angle \( Z \) are known.

![Figure 1](image)

Taking \( YZ \) as the base, draw a line from \( X \) meeting \( YZ \) in a right angle at \( H \). The length of this line, \( h \), is the height of the triangle. From triangle \( XZH \), we have \( \sin Z = h/y \) so that \( h = y \sin Z \).

Substituting this in \( A = \frac{1}{2}bh \) gives \( A = \frac{1}{2}xy \sin Z \).

The area of triangle \( XYZ \) is given by \( A = \frac{1}{2}xy \sin Z \).

Note that the angle \( Z \) must be the one between the sides of length \( x \) and \( y \).

Example 1

Find the area of triangle \( PQR \) in Figure 2 correct to one decimal place.

![Figure 2](image)

**Solution**

Since \( Q \) is the known angle, the area of the triangle is
\[
\frac{1}{2} \times PQ \times QR \times \sin Q = \frac{1}{2} \times 7.2 \times 6.3 \times \sin 40^\circ \\
= 14.6 \text{ cm}^2 \text{ (to 1 d.p.)}.
\]