**Refresh Algebraic fractions**

When dealing with algebraic expressions it is often easier if they are rewritten in (rearranged in or manipulated into) different forms. This is particularly true of expressions involving fractions or roots. Being able to recognise such rearrangement possibilities can in some contexts make handling expressions much simpler.

**Example 1**
(a) Rearrange \( \frac{p+q}{pq} \) as the sum of two fractions and simplify.
(b) Expand, simplify or otherwise manipulate the following into a single fraction.
\[
\frac{3}{\sqrt{x}} + \sqrt{x}.
\]

**Solution**
(a) \[
\frac{p+q}{pq} = \frac{p}{pq} + \frac{q}{pq} = \frac{1}{q} + \frac{1}{p}
\]
(b) \[
\frac{3}{\sqrt{x}} + \sqrt{x} = \frac{3}{\sqrt{x}} + \sqrt{x} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{3 + x \sqrt{x}}{\sqrt{x}} = \frac{3 + x}{\sqrt{x}}
\]

**Algebraic fractions (rational functions)**

Fractions involving expressions in \( x \) should be manipulated according to exactly the same rules as those for numerical fractions. For example, to combine
\[
\frac{1}{x+1} - \frac{1}{x+3},
\]
we use a common denominator – in this case \((x+1)(x+3)\) – and we begin by writing both fractions with this denominator.

**Example 2**
Simplify \( \frac{1}{x+1} - \frac{1}{x+3} \).

**Solution**
We combine the terms in the following way.
\[
\frac{1}{x+1} - \frac{1}{x+3} = \frac{x+3}{(x+1)(x+3)} - \frac{x+1}{(x+1)(x+3)} = \frac{(x+3) - (x+1)}{(x+1)(x+3)} = \frac{x+3-x-1}{(x+1)(x+3)} = \frac{2}{(x+1)(x+3)}
\]