S111 to MST124 - additional mathematics support

Chapter 2  Algebra
Contents

2 Algebra 3
  2.1 Collecting like terms 3
  2.2 Simplifying single terms 7
  2.3 Simplifying two or more terms 10
  2.4 Algebraic fractions 12
  2.5 Equivalent algebraic fractions 14
  2.6 Adding and subtracting algebraic fractions 16
  2.7 Multiplying and dividing algebraic fractions 18
  2.8 Multiplying out brackets 21
  2.9 Finding common factors 27
  2.10 Taking out common factors 29

Solutions and comments on Activities 34
2 Algebra

2.1 Collecting like terms

In this subsection you’ll learn the first of several useful techniques for simplifying expressions: collecting like terms.

Let’s look first at how it works with numbers. If you have 2 batches of 4 dots, and another 3 batches of 4 dots, then altogether you have

2 + 3 batches of 4 dots, that is, 5 batches of 4 dots.

This is shown in Figure 1, and you can express it by writing

\[ 2 \times 4 + 3 \times 4 = 5 \times 4. \]

Of course, this doesn’t work just with batches of four dots. For example, Figure 2 illustrates that

\[ 2 \times 7 + 3 \times 7 = 5 \times 7. \]

In fact, no matter what number \( a \) is,

\[ 2a + 3a = 5a. \]

This gives you a way to simplify expressions that contain a number of batches of something, added to another number of batches of the same thing. For example, consider the expression

\[ 5bc + 4bc. \]

Adding 4 batches of \( bc \) to 5 batches of \( bc \) gives 9 batches of \( bc \):

\[ 5bc + 4bc = (5 + 4)bc = 9bc. \]

Terms that are ‘batches of the same thing’ are called like terms. For terms to be like terms, the letters and the powers of the letters in each term must be the same. So, for example,

\[ 7\sqrt{A} \text{ and } 3\sqrt{A} \text{ are like terms because they are both terms in } \sqrt{A}; \]
\[ 2x^2 \text{ and } -0.5x^2 \text{ are like terms because they are both terms in } x^2. \]

However,

\[ 5c \text{ and } 4c^2 \text{ are not like terms because } 5c \text{ is a term in } c \text{ and } 4c^2 \text{ is a term in } c^2. \]

Activity 1  Identifying like terms

Which of the following are pairs of like terms?

(a) \( 3b \) and \( 6b^2 \)  \hspace{1cm} (b) \( 5D \) and \( 5D \)  \hspace{1cm} (c) \( z \) and \( -z \)  \hspace{1cm} (d) \( 3 \) and \( 2m \)

Like terms can always be collected in a similar way to the examples above: you just add the coefficients (including negative ones). You can add any number of like terms.
Example 1 Collecting like terms

Simplify the following expressions.
(a) \(12m + 15m - 26m\)  
(b) \(0.5XY^2 + 0.1XY^2\)  
(c) \(5p - p\)  
(d) \(\frac{7}{9}d - 2d\)

Solution
(a) \(12m + 15m - 26m = (12 + 15 - 26)m = m\)
(b) \(0.5XY^2 + 0.1XY^2 = (0.5 + 0.1)XY^2 = 0.6XY^2\)
(c) \(5p - p = 5p - 1p = (5 - 1)p = 4p\)
(d) \(\frac{7}{9}d - 2d = \left(\frac{7}{9} - 2\right)d = \left(\frac{7}{9} - \frac{18}{9}\right)d = -\frac{5}{3}d\)

Notice that in the solution to Example 1(d), the fractional coefficients were not converted to approximate decimal values. In algebra you should work with exact numbers, such as \(\frac{7}{9}\) and \(\sqrt{5}\), rather than decimal approximations, wherever possible. However, if you’re using algebra to solve a practical problem, then you may have to use decimal approximations.

Activity 2 Collecting like terms

Simplify the following expressions.
(a) \(8A + 7A\)  
(b) \(-5d + 8d - 2d\)  
(c) \(-7z + z\)  
(d) \(1.4pq + 0.7pq - pq\)  
(e) \(\frac{1}{4}n^2 - \frac{1}{3}n^2\)

It’s easier to spot like terms if you make sure that all the letters in each term are written in alphabetical order. For example, \(5st\) and \(2ts\) are like terms – this is easier to see if you write the second one as \(2st\).

Example 2 Recognising like terms

Simplify the following expressions.
(a) \(5st + 2ts\)  
(b) \(-6q^2rp + 4prq^2\)

Solution
(a) \(5st + 2ts = 5st + 2st = 7st\)
(b) \(-6q^2rp + 4prq^2 = -6pq^2r + 4pq^2r = -2pq^2r\)

The lower- and upper-case versions of the same letter are different symbols in mathematics. So, for example, \(4y\) and \(9Y\) are not like terms.

Any two constant terms are like terms. They can be collected using the normal rules of arithmetic.
**Activity 3  Recognising like terms**

Which of the following are pairs of like terms?

(a) $2ab$ and $5ab$
(b) $-2rst$ and $20rst$
(c) $2xy$ and $-3yx$
(d) $4ac^2$ and $9ac^2$
(e) $abc$ and $cba$
(f) $8c^2d$ and $9d^2c$
(g) $2A^2$ and $10a^2$
(h) $3fh$ and $3gh$
(i) $22$ and $-81$

Often an expression contains some like terms and some unlike terms. You can simplify the expression by first changing the order of its terms so that the like terms are grouped together, and then collecting the like terms. This leaves an expression in which all the terms are unlike, which can’t be simplified any further. Here’s an example.

**Example 3  Collecting more like terms**

Simplify the following expressions.

(a) $2a + 5b - 7a + 3b$
(b) $12 - 4pq - 2q + 1 - qp - 2$

**Solution**

(a) Group the like terms, then collect them.

\[
2a + 5b - 7a + 3b = 2a - 7a + 5b + 3b = -5a + 8b
\]

(b) Write $qp$ as $pq$, group the like terms, then collect them.

\[
12 - 4pq - 2q + 1 - qp - 2 = 12 - 4pq - 2q + 1 - pq - 2 = 12 + 1 - 2 - 4pq - pq - 2q = 11 - 5pq - 2q
\]

The terms in the final expression can be written in any order. For example, an alternative answer is $11 - 2q - 5pq$.

**Activity 4  Collecting more like terms**

Simplify the following expressions.

(a) $4A - 3B + 3C + 5A + 2B - A$
(b) $-8v + 7 - 5w - 2v - 8$
(c) $20y^2 + 10xy - 10y^2 - 5y - 5xy$
(d) $-4ef + 8e^2f + 10fe - 3f^2e$
(e) $\frac{1}{2}a + \frac{1}{3}b + 2a + \frac{1}{4}b$

You may find it helpful to mark the terms before rearranging them.
As you become more used to working with expressions, you’ll probably find that you can collect like terms without grouping them together first. The worked examples in the course will usually do this, and you should do so too, as soon as you feel comfortable with it.

Sometimes when you collect two or more like terms, you find that the result is zero – that is, the terms cancel each other out. Here’s an example.

**Example 4  Terms that cancel out**

Simplify the expression

\[ M + 2N + 3M - 2N. \]

**Solution**

\[ M + 2N + 3M - 2N = 4M + 0 = 4M \]

In the example above, \(2N\) is added and then subtracted, and the addition and subtraction cancel each other out.

**Activity 5  Terms that cancel out**

Simplify the following expressions.

(a) \(2a^3 - 3a - 2a^3 - 3a\)

(b) \(2m + n - 5m + 2n + 3m\)

(c) \(b + 2b + 3b - 6b\)

**Activity 6  Collecting like terms in a formula**

A primary school parents’ group is organising an outing to a children’s activity centre, for \(c\) children and \(a\) adults, travelling by train.

(a) The cost of a ticket for the activity centre is £10 for a child and £2 for an adult. Find a formula for \(A\), where £\(A\) is the total cost of admission to the activity centre for the group.

(b) The cost of a return train ticket to get to the activity centre is £7 for a child and £14 for an adult. Find a formula for \(T\), where £\(T\) is the total cost of travel for the group.

(c) By adding your answers to parts (a) and (b), find a formula for \(C\), where £\(C\) is the total cost of the trip for the group.

(d) Simplify the formula found in part (c) by collecting like terms, if you haven’t already done so in part (c).

(e) Use the formula found in part (d) to find the cost of the trip for 22 children and 10 adults.

In this section you have been introduced to some terminology used in algebra, and you have learned the algebraic technique of collecting like
Simplifying terms

Sometimes the terms in an expression need to be simplified, to make the expression easier to work with, and to make it easy to recognise any like terms. You’ll learn how to do that in this section. We begin by looking at terms individually, and later in the section we consider expressions with more than one term.

2.2 Simplifying single terms

As you’ve seen, if a term consists of numbers and letters all multiplied together, then it should be written with the coefficient first, followed by the letters. It’s often useful to write the letters in alphabetical order – for example, $3B^2DA$ as $3AB^2D$ – as this can help you to identify like terms in a complicated expression. This is usually done in this course. (However, there are some contexts where tradition requires a non-alphabetical order.)

If a term includes a letter multiplied by itself, then index notation should be used. For example,

- $p \times p$ should be simplified to $p^2$,
- $p \times p \times p$ should be simplified to $p^3$.

Example 5  Simplifying terms

Write the following terms in their shortest forms.

(a) $3 \times c \times g \times 4 \times b$  (b) $b \times a \times 5 \times b \times b$

**Solution**

(a) $3 \times c \times g \times 4 \times b = 12bcg$
(b) $b \times a \times 5 \times b \times b = 5ab^3$

When you simplify a term you should normally use index notation only for letters and not for numbers. For example,

- $3 \times 3 \times a$ should be simplified to $9a$, not $3^2a$.

Activity 7  Using index notation

Write the following terms in their shortest forms.

(a) $y \times z \times 6 \times x \times 4$  (b) $7p \times 2qr$  (c) $QR \times G \times 5F$
(d) $2 \times a \times a \times 3 \times a$  (e) $m \times n \times m \times 4$
(f) $5y \times 2yx$  (g) $4AB \times 4AB$
In Topic 1 of S111 you met the index law
\[ a^m \times a^n = a^{m+n} \].

If a term contains more than one power of the same letter, multiplied together, then the indices need to be added. For example,
\[ x^3 \times x^2 = x^5 \].

Remember that \( x \) is the same as \( x^1 \); for example,
\[ x \times x^7 = x^8 \].

**Example 6  Multiplying powers**

Write the following term in its shortest form:
\[ 2A^5B \times 3A^4B^7 \].

**Solution**
\[ 2A^5B \times 3A^4B^7 = 2 \times 3 \times A^{5+4}B^{1+7} = 6A^9B^8 \]

**Activity 8  Multiplying powers**

Write the following terms in their shortest forms.
(a) \( 8P^8 \times 5P \)  
(b) \( 2c^{10}d^3 \times 2c^2d^3 \)

If a term consists of numbers and letters multiplied together, and some of these have minus signs, then the overall sign of the term can be worked out using the rules below. The rest of the term can be simplified in the usual way.

When multiplying or dividing:
- two signs the same give a plus sign;
- two different signs give a minus sign.

Here are some examples to illustrate these rules.
\[ 2 \times (-3) = -6, \quad (-2) \times (-3) = 6, \]
\[ a \times (-b) = -ab, \quad (-a) \times (-b) = ab. \]

**Example 7  Simplifying terms involving minus signs**

Write the following terms in their shortest forms.
(a) \( 4q \times (-2p) \)  
(b) \( -B^3 \times (-5B) \)  
(c) \( -a \times (-b) \times (-a) \)

**Solution**
(a) ☐ A positive times a negative gives a negative. ☐
\[ 4q \times (-2p) = -4q \times 2p = -8pq \]

(b) ☐ A negative times a negative gives a positive. ☐
\[ -B^3 \times (-5B) = +B^3 \times 5B = +5B^4 \]
\[ = 5B^4 \]
The first negative times the second negative gives a positive, then that positive times the third negative gives a negative. 

\[-a \times (-b) \times (-a) = -a \times b \times a = -a^2b\]

The overall sign is found in the same way as when you multiply several negative numbers together. See Topic 1 of S111.

The box below summarises how to simplify a term.

**Strategy** To simplify a term

1. Find the overall sign and write it at the front.
2. Simplify the rest of the coefficient and write it next.
3. Write the letters in alphabetical order (usually), using index notation as appropriate.

If the coefficient of a term is 1 or \(-1\), then the 1 should be omitted. For example,

1\(xy\) should be simplified to \(xy\),

and

\(-1c^2\) should be simplified to \(-c^2\).

In the next activity, try to simplify the terms in a single step, using the strategy above. This skill will be helpful later, when you learn to simplify more complicated expressions.

**Activity 9** Simplifying terms involving minus signs

Write the following terms in their shortest forms.

(a) \(9X \times (-XY)\)  
(b) \(3s \times \frac{4}{r}\)  
(c) \(-3a^3 \times (-4a^4)\)  
(d) \(-2pq \times (-3pq^2)\)  
(e) \(-0.5g \times 2f^5\)  
(f) \(-a \times b \times (-c) \times (-d)\)  
(g) \((-x) \times (-y) \times (-x^2) \times (-4y)\)  
(h) \((-3cd)^2\)  
(i) \(-(3cd)^2\)

Hint for parts (h) and (i): something\(^2\) = the something × the something.

Expressions can contain terms of the form 

\[+(-\text{something}) \quad \text{or} \quad -(\text{-something}).\]

These should be simplified by using the following facts.

- Adding the negative of something is the same as subtracting the something.
- Subtracting the negative of something is the same as adding the something.

You saw these rules for numbers in Topic 1 of S111.

Here are some examples:

\[+(-8) = -8, \quad -(-5) = +5 = 5,\]

\[+(-2M^2) = -2M^2, \quad -(-x) = +x = x.\]
Also, any unnecessary brackets in a term should usually be removed. For example:

\[ + (pq) = +pq = pq, \]
\[ -(7z) = -7z. \]

The brackets in these terms aren’t needed, because by the BIDMAS rules multiplication is done before addition and subtraction.

Before removing unnecessary brackets, check carefully that they really are unnecessary! If you’re not sure, leave the brackets in.

**Activity 10  Simplifying the signs of terms**

Write the following terms in their shortest forms.

(a) \[ + (−ab) \]
(b) \[ − (−6x^2) \]
(c) \[ − (2M^4) \]
(d) \[ + (−7y) \]
(e) \[ + (5p) \]
(f) \[ − (−\frac{3}{5}n) \]

**2.3 Simplifying two or more terms**

So far in this section you’ve been simplifying single terms. To simplify an expression with two or more terms, you need to simplify each term individually, in the way that you’ve seen. Before you can do that, you need to identify which bits of the expression belong to which term. The easiest way to do that is to use the following fact.

Each term after the first starts with a plus or minus sign that isn’t inside brackets.

**Example 8  Identifying terms**

Mark the terms in the following expressions.

(a) \[ −2a − (−5a^2) + (−4a) \]
(b) \[ 2x \times 4xy − 2y \times (−5x) \]

**Solution**

(a) \[ \bigcirc \text{ Begin by marking the start of the first term. } \bigcirc \]
\[ −2a − (−5a^2) + (−4a) \]

\[ \bigcirc \text{ Extend the line under the first term until you reach a plus or minus sign that isn’t inside brackets. That’s the start of the next term. } \bigcirc \]
\[ −2a − (−5a^2) + (−4a) \]

\[ \bigcirc \text{ Extend the line under the second term until you reach a plus or minus sign that isn’t inside brackets. That’s the start of the next term. } \bigcirc \]
\[ −2a − (−5a^2) + (−4a) \]

\[ \bigcirc \text{ Extend the line under the third term until you reach a plus or minus sign that isn’t inside brackets. This time you don’t reach one } \bigcirc \]
you just reach the end of the expression. So this expression has three terms, as marked.

\[-2a - (-5a^2) + (-4a)\]

(b) Mark the start of the first term.

\[2x \times 4xy - 2y \times (-5x)\]

When you reach a plus or minus sign that isn’t inside brackets, that’s the start of the next term.

\[2x \times 4xy - 2y \times (-5x)\]

You don’t reach another plus or minus sign that isn’t inside brackets, so this expression has two terms.

\[2x \times 4xy - 2y \times (-5x)\]

Example 8 showed you how to carry out the first step in the following strategy. The other two steps use techniques that you’ve seen already.

**Strategy** *To simplify an expression with more than one term*

1. Identify the terms. Each term after the first starts with a plus or minus sign that isn’t inside brackets.
2. Simplify each term, using the strategy on page 9. Include the sign (plus or minus) at the start of each term.
3. Collect any like terms.

In the next example, this strategy is used to simplify the expressions in Example 8.

**Example 9  Simplifying expressions with more than one term**

Simplify the following expressions.

(a) \(-2a - (-5a^2) + (-4a)\)  
(b) \(2x \times 4xy - 2y \times (-5x)\)

**Solution**

(a) First identify the terms. Then simplify each term individually.

Finally, collect like terms.

\[-2a - (-5a^2) + (-4a) = -2a + 5a^2 - 4a\]

\[= 5a^2 - 6a\]

This could also be written as \(-6a + 5a^2\).

(b) Identify the terms, then simplify each term individually. Finally, check for like terms – there are none here.

\[2x \times 4xy - 2y \times (-5x) = 8x^2y + 10xy\]

In the next activity, begin each part by marking the terms, as shown in Example 8. As you become more used to manipulating expressions, you’ll probably find that you can identify and simplify the terms without needing to mark them.
Activity 11  Simplifying expressions

Simplify the following expressions.

(a) \(5m \times 2m - 2n \times n^2\)  
(b) \(3p \times 2q + 2r \times (-7p)\)  
(c) \(2P - (-3Q) + (-P) + (2Q)\)  
(d) \(3 \times (-2a) - 1c^2 + 9ac\)  
(e) \(4s \times \frac{1}{2}rst - 2(-s)\)  
(f) \(-5xy + (-3y \times x^2) - (-y^2)\)  
(g) \(-3r \times (-2r) - (-2r \times r) + (r^2 \times 9)\)

2.4 Algebraic fractions

It’s usually best to use fraction notation, rather than a division sign, to indicate division in algebraic expressions. For example, the expression

\[
a + b \div c \quad \text{(1)}
\]

can be written as

\[
a + \frac{b}{c}. \quad \text{(2)}
\]

This makes it easy to see which parts of the expression are divided by which. In expression (1), it’s just \(b\), not \(a + b\), that’s divided by \(c\), by the BIDMAS rules. This is clearer in expression (2).

Similarly, the expression

\[
(8a + 3) \div (2a) \quad \text{(3)}
\]
can be written in fraction notation as

\[
\frac{8a + 3}{2a}.
\]

The brackets can be omitted because the fraction notation makes it clear that the whole of \(8a + 3\) is divided by \(2a\).

Algebraic expressions written using fraction notation are called algebraic fractions. The expressions above and below the line in an algebraic fraction are called the numerator and denominator, respectively, just as they are for ordinary fractions.

When you write an algebraic fraction, you must make sure that the horizontal line extends to the full width of the numerator or denominator, whichever is the wider. For example,

\[
(8a + 3) \div (2a) \quad \text{should be written as} \quad \frac{8a + 3}{2a}, \quad \text{not} \quad \frac{8a + 3}{2a}. \quad \text{(4)}
\]

This is because the line acts as brackets for the numerator and denominator, as well as indicating division.

Try not to use division signs in expressions, or whenever you carry out algebraic manipulation, from now on; use fraction notation instead. However, occasionally it’s useful to use division signs in algebraic expressions, just as occasionally it’s useful to use multiplication signs.
Activity 12  Using fraction notation

Rewrite the following expressions using fraction notation.

(a) \((a + b) \div 3\)  \hspace{1em} (b) \(a + b \div 3\)  \hspace{1em} (c) \((x + 2) \div (y + 3)\)

(d) \((x + 2) \div y + 3\)  \hspace{1em} (e) \(x + 2 \div (y + 3)\)  \hspace{1em} (f) \(x + 2 \div y + 3\)

Activity 13  Working with fraction notation

Multiply out the brackets in the following expressions.

(a) \(6 \left(1 + \frac{h}{2}\right)\)  \hspace{1em} (b) \(6 \left(\frac{1 + h}{2}\right)\)

Hint for part (b): First write the expression in the form ‘number \times (1 + h)’.

There’s a technique, based on multiplying out brackets, that can be useful when you’re working with algebraic fractions. As with multiplying out brackets, this technique doesn’t necessarily simplify an expression; it just gives a different way of writing it. It applies to algebraic fractions where there’s more than one term in the numerator, such as

\[
\frac{2a - 5b + c}{3d}.
\]

(4)

Since dividing by something is the same as multiplying by its reciprocal, you can write this expression as

\[
\frac{1}{3d}(2a - 5b + c).
\]

You can then multiply out the brackets to give

\[
\frac{2a}{3d} - \frac{5b}{3d} + \frac{c}{3d}.
\]

(5)

If you compare expressions (4) and (5), you can see that the overall effect is that each term on the numerator has been individually divided by the denominator. This is called expanding the algebraic fraction.

Once an algebraic fraction has been expanded, it may be possible to simplify some of the resulting terms, as illustrated in the next example.

Example 10  Expanding an algebraic fraction

Expand the algebraic fraction \(\frac{10x + x^2 - 8}{x}\).

Solution

\[
\frac{10x + x^2 - 8}{x} = \frac{10x}{x} + \frac{x^2}{x} - \frac{8}{x} \\
= 10 + x - \frac{8}{x}
\]

Remember: \(\frac{x^2}{x} = \frac{x \times x}{x} = x\).
Remember that an algebraic fraction can be expanded only if it has more than one term in the numerator. The following fraction, which has more than one term in the denominator, can’t be expanded:

$$\frac{a}{2a + 5b - c}$$

### Activity 14 Expanding algebraic fractions

Expand the following algebraic fractions, and simplify the resulting expressions where possible.

(a) \( \frac{A - 6B}{3} \)  
(b) \( \frac{10z^2 + 5z - 20}{5} \)  
(c) \( \frac{3A^2 + A}{A} \)

### Manipulating algebraic fractions

Fractions that involve algebraic expressions are called algebraic fractions. Here are some examples:

\[
\frac{8ab}{c}, \quad \frac{1}{(x - 3)^2} \quad \text{and} \quad \frac{2x + 1}{x^2 + 1}.
\]

Algebraic fractions appear in many important formulas. For example,

\[
F = \frac{GmM}{r^2}
\]

is a formula for the force of gravitational attraction \( F \) between two objects that have masses \( m \) and \( M \), and are a distance \( r \) apart; here \( G \) is a constant, called the universal gravitational constant, measured in suitable units.

To be able to work with such formulas, you need to be able to manipulate algebraic fractions in a similar way to numerical fractions. For example, you should be able to add them and multiply them. In this section you will practise working with algebraic fractions, and in the next section you will meet some problems that are solved by manipulating algebraic fractions.

When you work with algebraic fractions, there is a risk of considering values of the variables for which the denominator of the fraction is 0. Since dividing by 0 is not allowed, you need to avoid using such values of the variables! For example, the formula above for the force \( F \) due to gravity cannot be applied with \( r = 0 \). In fact, \( r \) takes only positive values, since it represents a distance between two different objects.

### 2.5 Equivalent algebraic fractions

Recall from Topic 1 of S111 that two numerical fractions are equivalent if one can be obtained from the other by multiplying or dividing both the numerator and the denominator by the same number. For example,

\[
\frac{2}{6} \quad \text{is equivalent to} \quad \frac{1}{3}, \quad \text{because} \quad \frac{2}{6} = \frac{1 \times 2}{3 \times 2}.
\]
Similarly, two algebraic fractions are equivalent if one can be obtained from the other by multiplying or dividing both the numerator and the denominator by the same expression. For example,

\[
\frac{a}{b} \text{ is equivalent to } \frac{a(a + 1)}{b(a + 1)}.
\]

The process of simplifying a numerical fraction by dividing both the numerator and the denominator by the same number is called **cancelling** a common factor. For example, the fraction \(\frac{2}{6}\) can be simplified by cancelling the common factor 2:

\[
\frac{2}{6} = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}.
\]

Algebraic fractions can be simplified in the same way, by cancelling any common factors of the numerator and denominator. You can represent the cancellation in the usual way, by crossing out the expressions with common factors and writing the results of the cancellations nearby.

You can find any common factors of the numerator and denominator of an algebraic fraction by expressing the numerator and denominator as products of all their factors, if this is not done already. Here are some examples.

### Example 11  Simplifying algebraic fractions

Simplify each of the following algebraic fractions.

(a) \(\frac{a^2}{a^5}\)  
(b) \(\frac{60p^3q}{35p^5r}\)  
(c) \(\frac{2x^2 + 6x}{x^2 - 9}\)

**Solution**

(a) Divide top and bottom by \(a^2\).

\[
\frac{a^2}{a^5} = \frac{1}{a^3}
\]

Now top and bottom have no common factors, so the fraction can’t be simplified further.

(b) Divide top and bottom by 5 and \(p^3\).

\[
\frac{60p^3q}{35p^5r} = \frac{12}{7p^2r}
\]

(c) To find any common factors, factorise the top and bottom.

\[
\frac{2x^2 + 6x}{x^2 - 9} = \frac{2x(x + 3)}{(x - 3)(x + 3)} = \frac{2x}{x - 3}
\]

The denominator \(x^2 - 9\) is a difference of two squares.

In Example 11, the expressions that are factors of the denominators are all assumed to be non-zero. For example, in part (a) it is assumed that \(a \neq 0\). Similarly, in part (c) it is assumed that \(x - 3 \neq 0\) and \(x + 3 \neq 0\). Often, assumptions of this type are not stated explicitly, but you should be aware of them.
It is important to cancel only expressions that are factors of both the numerator and the denominator of a fraction. For example, in the fraction \[ \frac{2x^2 + 6x}{x^2 - 9}, \]
in Example 11, it would have been wrong to cancel the expression \(x^2\) in the numerator and denominator, because \(x^2\) is not a factor of the numerator or denominator. So before you cancel anything, check that it is a factor of both the top and the bottom!

Here are some algebraic fractions for you to simplify.

### Activity 15  Simplifying algebraic fractions

Simplify the following algebraic fractions.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>( \frac{x^4}{x^9} )</td>
</tr>
<tr>
<td>(b)</td>
<td>( \frac{20a^2b}{15ab^2} )</td>
</tr>
<tr>
<td>(c)</td>
<td>( \frac{x^2 + 6x}{3x^2} )</td>
</tr>
<tr>
<td>(d)</td>
<td>( \frac{u^2 - 4}{u^2 + 4u + 4} )</td>
</tr>
</tbody>
</table>

### 2.6 Adding and subtracting algebraic fractions

The rules for adding and subtracting algebraic fractions are the same as those for adding and subtracting numerical fractions, which were given in Topic 1 of S111.

Here are some numerical examples. If the denominators are the same, then you just add or subtract the numerators:

\[ \frac{3}{5} - \frac{2}{5} = \frac{1}{5}. \]

If the denominators are different, then you need to write the fractions with a common denominator before you can add or subtract them. For example, \( \frac{2}{5} \) and \( \frac{3}{8} \) can be added or subtracted by using the common denominator \( 5 \times 8 = 40 \), as follows:

\[ \frac{2}{5} + \frac{3}{8} &= \frac{2 \times 8}{5 \times 8} + \frac{3 \times 5}{8 \times 5} = \frac{16}{40} + \frac{15}{40} = \frac{31}{40}, \]
\[ \frac{2}{5} - \frac{3}{8} &= \frac{2 \times 8}{5 \times 8} - \frac{3 \times 5}{8 \times 5} = \frac{16}{40} - \frac{15}{40} = \frac{1}{40}. \]

The idea of finding a common denominator works for algebraic fractions too – the general strategy is given below.

### Strategy  To add or subtract algebraic fractions

1. Make sure that the fractions have a common denominator – if necessary, rewrite each fraction as an equivalent fraction.
2. Add or subtract the numerators.
3. Simplify the answer by cancelling wherever possible.
Here are some examples to illustrate this strategy.

**Example 12  Adding and subtracting algebraic fractions**

Write each of the following expressions as a single algebraic fraction.

(a) \( \frac{3}{x} - \frac{2}{x} \)  (b) \( \frac{3}{a+1} + \frac{2}{a+1} - \frac{1}{a+1} \)  (c) \( \frac{a}{x} + \frac{b}{y} \)

**Solution**

(a) The denominators are the same, so subtract the numerators.
\[
\frac{3}{x} - \frac{2}{x} = \frac{3-2}{x} = \frac{1}{x}
\]

(b) The denominators are the same, so add and subtract the numerators.
\[
\frac{3}{a+1} + \frac{2}{a+1} - \frac{1}{a+1} = \frac{3+2-1}{a+1} = \frac{4}{a+1}
\]

(c) The product of the denominators is \(xy\), so use this as a common denominator.
\[
\frac{a}{x} + \frac{b}{y} = \frac{ay}{xy} + \frac{bx}{xy} = \frac{ay+bx}{xy}
\]

As illustrated in Example 12(c), when you want to add or subtract algebraic fractions that do not have a common denominator, you can always obtain a common denominator by multiplying together the denominators of the given fractions.

**Activity 16  Adding and subtracting algebraic fractions**

Write each of the following expressions as a single algebraic fraction.

(a) \( \frac{6}{y} + \frac{1}{y} \)  (b) \( \frac{x}{a^2} + \frac{y}{a^2} - \frac{z}{a^2} \)  (c) \( \frac{a}{2x} - \frac{b}{3y} \)  (d) \( \frac{1}{3x} - \frac{2}{x+3} \)

Although you can always obtain a common denominator by multiplying together the denominators of the given fractions, there is sometimes a simpler common denominator. In the example
\[
\frac{2}{a} + \frac{1}{a^2},
\]
the product of the denominators is \(a \times a^2 = a^3\). But a simpler common denominator is \(a^2\), because \(a^2\) is a multiple of both \(a\) and \(a^2\). Using this common denominator gives
\[
\frac{2}{a} + \frac{1}{a^2} = \frac{2a}{a^2} + \frac{1}{a^2} = \frac{2a+1}{a^2}.
\]

If you had used the common denominator \(a^3\), then the result would have been the same, but only after cancelling a common factor of \(a\) from the numerator and the denominator.

To find the simplest common denominator, you should factorise the denominators of the given fractions, if possible, and then choose the simplest expression that is a multiple of each denominator. For example,
to write \( \frac{x}{xy + y^2} + \frac{y}{x^2 + xy} \)
as a single algebraic fraction, you should factorise the denominators to give \( \frac{x}{y(x + y)} + \frac{y}{x(x + y)} \)
and then choose as your common denominator \( xy(x + y) \). So
\[
\frac{x}{xy + y^2} + \frac{y}{x^2 + xy} = \frac{x}{y(x + y)} + \frac{y}{x(x + y)} = \frac{x}{xy(x + y)} + \frac{y^2}{xy(x + y)} = \frac{x^2 + y^2}{xy(x + y)}.
\]
Also, when working with fractions you sometimes need to make an expression into a fraction in order to combine it with another fraction. For example, to express \( a + \frac{2}{b} \)
as a single fraction, you can write \( a \) as the fraction \( a/1 \), and then choose the common denominator to be \( b \):
\[
a + \frac{2}{b} = a \frac{1}{1} + \frac{2}{b} = \frac{ab}{b} + \frac{2}{b} = \frac{ab + 2}{b}.
\]

**Activity 17 Choosing denominators**

Write each of the following expressions as a single algebraic fraction.
(a) \( \frac{5}{x^4} - \frac{4}{x^2} \)  
(b) \( \frac{1}{x^2 + x} - \frac{1}{x + 1} \)  
(c) \( a + \frac{a}{b + 3} \)

**2.7 Multiplying and dividing algebraic fractions**

The rules for multiplying and dividing algebraic fractions are the same as those for multiplying and dividing numerical fractions, which were given in Topic 1 of S111.

Here are some numerical examples. To multiply two numerical fractions, multiply the numerators together and multiply the denominators together:
\[
\frac{1}{5} \times \frac{3}{5} = \frac{1 \times 3}{5 \times 5} = \frac{3}{25}.
\]
Remember that to obtain the reciprocal of a fraction, you swap the numerator and denominator; in other words, you ‘turn the fraction upside down’.

To divide by a numerical fraction, multiply by its reciprocal:
\[
\frac{1}{3} \div \frac{1}{5} = \frac{1}{3} \times \frac{5}{1} = \frac{5}{3}.
\]
Strategy  To multiply or divide algebraic fractions

- To multiply two algebraic fractions, multiply the numerators together and multiply the denominators together:
  \[
  \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}.
  \]

- To divide one algebraic fraction by another, multiply the first fraction by the reciprocal of the second fraction:
  \[
  \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}.
  \]

In each case, you should cancel any common factors that appear.

The following examples illustrate this strategy.

Example 13  Multiplying algebraic fractions

Write each of the following expressions as a single algebraic fraction, simplifying your answer if possible.

(a) \[
\frac{a}{2x} \times \frac{b}{3y} = \frac{ab}{6xy}
\]

Solution

(a) \[
\frac{a}{2x} \times \frac{b}{3y} = \frac{a \times b}{2x \times 3y} = \frac{ab}{6xy}
\]

(b) \[
\frac{3x}{5y} \times \frac{y(y + 1)}{x^2} = \frac{3x \times y(y + 1)}{5y \times x^2} = \frac{3x \times y(y + 1)}{5y \times x^2} \times \frac{1}{1} = \frac{3(y + 1)}{5x}
\]

Example 14  Dividing algebraic fractions

Write each of the following expressions as a single algebraic fraction, simplifying your answer if possible.

(a) \[
\frac{a}{2x} \div \frac{b}{3y} = \frac{a \times b}{2x \times 3y} = \frac{a \times b}{2x \times 3y} = \frac{3ay}{2bx}
\]

Solution

(a) \[
\frac{a}{2x} \div \frac{b}{3y} = \frac{a}{2x} \times \frac{3y}{b} = \frac{a \times 3y}{2x \times b} = \frac{3ay}{2bx}
\]

(b) \[
\frac{3x}{5y} \div \frac{y(y + 1)}{x^2} = \frac{3x}{5y} \times \frac{x^2}{y(y + 1)} = \frac{3x \times x^2}{5y \times y(y + 1)} = \frac{3x^3}{5y^2(y + 1)}
\]

There are also some special cases of multiplying and dividing fractions. These arise when one of the expressions being multiplied or divided is not written as a fraction.
For example, you can do the following multiplication and division by writing $3b^2$ as a fraction with denominator 1:

$$3b^2 \times \frac{1}{a} = \frac{3b^2}{1} \times \frac{1}{a} = \frac{3b^2}{a}$$

and

$$\frac{b}{b + 1} \div 3b^2 = \frac{b}{b + 1} \div \frac{3b^2}{1} = \frac{b}{b + 1} \times \frac{1}{3b^2} = \frac{1}{3b(b + 1)}.$$

Here and from now on, the crossing out for any cancellation is not shown, but you may choose to include it if you find it helpful.

Here are some examples of multiplying and dividing algebraic fractions for you to try.

**Activity 18  Multiplying and dividing fractions**

Write each of the following expressions as a single algebraic fraction, simplifying your answer if possible.

(a) $\frac{p^2}{q^2} \times \frac{p}{q}$  (b) $\frac{p^2}{q^2} \div \frac{p}{q}$  (c) $\frac{9ax^2}{b} \times \frac{b^3}{4xy^2}$

(d) $\frac{9ax^2}{b} \div \frac{b^3}{4xy^2}$  (e) $8u^2 \times \frac{2}{u^2 + u}$

To end this section, here is a practical application of dividing fractions.

**Activity 19  Finding the ratio of two forces**

In this activity, the mass of the Earth is $M$ kg and its radius is $R$ km, and the mass of the Moon is $m$ kg and its radius is $r$ km. The gravitational force acting on an object of mass 1 kg is given by the formulas

$$F = \frac{GM}{R^2} \text{ on the surface of the Earth}$$

and

$$f = \frac{Gm}{r^2} \text{ on the surface of the Moon},$$

where $G$ is a constant.

(a) Express the ratio $F/f$ as an algebraic fraction involving $M$, $m$, $R$ and $r$.

(b) Assuming that $M = 80m$ and $R = 4r$, deduce that $F = 5f$. 

The actual values, to two significant figures, are as follows:

$M = 6.0 \times 10^{24}$ kg,

$R = 6400$ km,

$m = 7.4 \times 10^{22}$ kg,

$r = 1700$ km.

The data above show that these equations are approximately correct.
Part (b) of Activity 19 shows that the gravitational force acting on an object on the surface of the Earth is approximately five times greater than the gravitational force acting on the same object on the surface of the Moon. One dramatic consequence of this fact is that an astronaut can jump five times higher on the Moon than on the Earth! However, for a direct comparison the astronaut would have to wear the same spacesuit on Earth as on the Moon.

**Brackets**

In this section you’ll learn how to rewrite expressions that contain brackets as expressions without brackets, and you’ll also see some applications of Algebra.

### 2.8 Multiplying out brackets

Any expression that contains brackets, such as

\[ 8a + 3b(b - 2a) \]

or

\[ (2m + 3n) - (m + n - 3r) \]

can be rewritten without brackets. To see how to do this, let’s start by looking at an expression that involves only numbers:

\[ (2 + 3) \times 4. \]

When you learned to collect like terms, you saw that 2 + 3 batches of 4 dots is the same as 2 batches of 4 dots plus 3 batches of 4 dots, as illustrated in Figure 3. So \((2 + 3) \times 4\) is equivalent to

\[ 2 \times 4 + 3 \times 4. \]

Here an expression containing brackets has been rewritten as an expression without brackets:

\[ (2 + 3) \times 4 = 2 \times 4 + 3 \times 4. \]

It’s usual to write numbers in front of brackets, so let’s write the 4 first in each multiplication:

\[ 4(2 + 3) = 4 \times 2 + 4 \times 3. \]

Here you can see how to rewrite an expression with brackets as one without brackets: you multiply each of the numbers inside the brackets individually by the number outside the brackets. This is called **multiplying out the brackets**, **expanding the brackets**, or simply **removing the brackets**. The number outside the brackets is called the **multiplier**. Here’s another example, with multiplier 7.

\[ 7(1 + 5) = 7 \times 1 + 7 \times 5 \]

Multiplying out the brackets can be particularly helpful for expressions that contain letters. The rule above applies in just the same way.
Strategy  To multiply out brackets

Multiply each term inside the brackets by the multiplier.

Here are two examples, with multipliers $a$ and 3, respectively.

$$a(b + c) = ab + ac$$

$$3(p + q^2 + r) = 3p + 3q^2 + 3r$$

It doesn’t matter whether the multiplier is before or after the brackets. Here’s an example of multiplying out where the multiplier is after the brackets:

$$(x + y)z = xz + yz.$$ 

If you prefer the multiplier to be before the brackets, then you can change the order before multiplying out. For example,

$$(x + y)z = z(x + y) = zx + zy = xz + yz.$$ 

When you multiply out brackets, you often need to simplify the resulting terms, as illustrated in the next example.

Example 15  Multiplying out brackets

Multiply out the brackets in the following expression:

$$2a(3a + 2b).$$

Solution

$$2a(3a + 2b) = 2a \times 3a + 2a \times 2b$$

$$= 6a^2 + 4ab$$

Activity 20  Multiplying out brackets

Multiply out the brackets in the following expressions.

(a) $3p(pq + 4)$    (b) $x^2(x + 2y)$

Once you’re familiar with how to multiply out brackets, it’s usually best to simplify the terms as you multiply out, instead of first writing down an expression containing multiplication signs. This leads to tidier expressions and fewer errors.

For example, if you look at the expression in Example 15,

$$2a(3a + 2b),$$

you can see that when you multiply out the brackets, the first term will be $2a$ times $3a$. You simplify this to $6a^2$, using the strategy of first finding the sign, then the rest of the coefficient and then the letters, and write it down. Then you see that the second term is $2a$ times $-2b$, simplify this to $-4ab$, and write it down after the first term. This gives

$$2a(3a + 2b) = 6a^2 + 4ab.$$
Try this shorter form of working in the following activity.

**Activity 21  Multiplying out brackets efficiently**

Multiply out the brackets in each of the following expressions.

(a) $f(e + 5g)$  
(b) $5(2A + B)$  
(c) $3c(4c + 2d)$  
(d) $(a - b)c^2$  
(e) $2y(x + 2y + 4z)$  
(f) $2\left(\frac{1}{2}A^2 + \frac{3}{2}\right)$  
(g) $a(x + y)z$  
(h) $2b(b^2 + 2b^4)$

Simplifying the terms at the same time as multiplying out is particularly helpful when some of the terms inside the brackets, or the multiplier, have minus signs. For example, let’s multiply out the brackets in the expression 

$$3m(-2m + 3n - 6).$$

The first term is $3m$ times $-2m$, which simplifies to $-6m^2$. Working out the other terms in a similar way, we obtain

$$3m(-2m + 3n - 6) = -6m^2 + 9mn - 18m.$$

Here’s another example. In this one the multiplier has a minus sign.

**Example 16  Multiplying out brackets involving minus signs**

Multiply out the brackets in the following expression:

$$-a(b - a + 7).$$

**Solution**

$$-a(b - a + 7) = -ab + a^2 - 7a$$

The terms are

$-a \times b = -ab$,  
$-a \times (-a) = +a^2$,  
$-a \times 7 = -7a$.

**Activity 22  Multiplying out brackets involving minus signs**

Multiply out the brackets in the following expressions, simplifying where possible.

(a) $p(q - r)$  
(b) $7a(-4a + 3b)$  
(c) $6(0.2a - 0.3b + 1.4)$  
(d) $10\left(\frac{1}{2}n + \frac{1}{3}\right)$  
(e) $-3(x - 2y)$  
(f) $-b^2(-a + b)$

An expression containing brackets may have more than one term. For example, the expression

$$x(y + 1) + 2y(y + 3)$$

has two terms, each containing brackets, as follows:

$$x(y + 1) + 2y(y + 3).$$
An expression like this can be dealt with term by term, using a similar strategy to the one for simplifying expressions.

**Strategy** To multiply out brackets in an expression with more than one term

1. Identify the terms. Each term after the first starts with a plus or minus sign that isn’t inside brackets.
2. Multiply out the brackets in each term. Include the sign (plus or minus) at the start of each resulting term.
3. Collect any like terms.

While you’re learning to multiply out brackets in expressions with more than one term, you’ll probably find it helpful to mark the terms as you identify them. This should help you to avoid errors.

### Example 17 Expanding the brackets when there’s more than one term

Multiply out the brackets in the following expressions, simplifying where possible.

(a) \( x(y + 1) + 2y(y + 3) \)  
(b) \( 2r^2 - r(r - s) \)

**Solution**

(a) \( \Rightarrow \) Identify the terms. Multiply out the brackets. Then check for like terms – there are none here. \( \Rightarrow \)

\[
x(y + 1) + 2y(y + 3) = xy + x + 2y^2 + 6y
\]

(b) \( \Rightarrow \) Identify the terms. Multiply out the brackets. Collect like terms. \( \Rightarrow \)

\[
2r^2 - r(r - s) = 2r^2 - r^2 + rs = r^2 + rs
\]

### Activity 23 Expanding the brackets when there’s more than one term

Multiply out the brackets in the following expressions, simplifying where possible.

(a) \( f + g(f + h) \)  
(b) \( x - y(x + 2y) \)  
(c) \( 2p - 3q(-3p + q) \)  
(d) \( -2(a + b) + 4(a - b) \)  
(e) \( 2aE - 3E(-E - 5a) \)  
(f) \( (d - c)c - c^2 \)

Some expressions, such as

\(- (a + 2b - c),\)

contain brackets with just a minus sign in front.

You can remove these brackets by using the fact that a minus sign in front is just the same as multiplying by \(-1\):

\[
-(a + 2b - c) = -1(a + 2b - c) = -a - 2b + c.
\]
You can see that the overall effect is that the sign of each term in the brackets has changed.

An expression may also contain brackets with just a plus sign in front. These brackets can be removed by using the fact that a plus sign in front is just the same as multiplying by 1. For example, the expression

\[ 2x + (y - 3z) \]

can be simplified as follows:

\[
2x + (y - 3z) = 2x + 1(y - 3z)
\]

\[= 2x + y - 3z.\]

This time you can see that the effect is that all the signs in the brackets remain as they are.

Rather than introducing 1 or \( -1 \) into working, as was done above, it’s better to remember the following strategy.

**Strategy**  
To remove brackets with a plus or minus sign in front

- If the sign is plus, keep the sign of each term inside the brackets the same.
- If the sign is minus, change the sign of each term inside the brackets.

**Example 18  Plus and minus signs in front of brackets**

Remove the brackets in the following expressions.

(a) \(-(-P^2 + 2Q - 3R)\)  
(b) \(a + (2bc - d)\)

**Solution**

(a) \(-(-P^2 + 2Q - 3R) = +P^2 - 2Q + 3R\)

\[\begin{align*}
&= P^2 - 2Q + 3R
\end{align*}\]

(b) \(a + (2bc - d) = a + 2bc - d\)

**Activity 24  Plus and minus signs in front of brackets**

Remove the brackets in each of the following expressions, and simplify them where possible.

(a) \(-(4f - g^3)\)  
(b) \(-(-x + 7y - 8z + 6)\)  
(c) \(2(a - b) + (c - 2d)\)  
(d) \(r + (-2s - r)\)  
(e) \(-A + B - (-3A + 4B)\)  
(f) \(-(-t - w) + (-t + w)\)  
(g) \(-(L + 2M) - (-M)\)
Some expressions, such as 
\[(x + 2)(x - 5),\]
contain two, or even more, pairs of brackets multiplied together. You’ll learn how to multiply out brackets like these in Chapter 4.

You’ve seen that you should usually write expressions in the simplest way you can. For example, you should write
\[x + 2x + 3x \quad \text{as} \quad 6x.\]

The second form of this expression is clearly simpler than the first:

- it’s shorter and easier to understand, and
- it’s easier to evaluate for any particular value of \(x\).

These are the attributes to aim for when you try to write an expression in a simpler way.

However, sometimes it’s not so clear that one way of writing an expression is better than another. For example,
\[x(x + 1) \quad \text{is equivalent to} \quad x^2 + x.\]

Both these forms are reasonably short, and both are reasonably easy to evaluate. So this expression doesn’t have a simplest form.

The same is true of many other expressions. You should try to write each expression that you work with in a reasonably simple way, but often there’s no ‘right answer’ for the simplest form. One form might be better for some purposes, and a different form might be better for other purposes.

In particular, multiplying out the brackets in an expression doesn’t always simplify it.

**Common factors**

The new technique that you need for rearranging equations is called *taking out common factors*, and is the reverse of multiplying out brackets. For example, if you have the expression \(x^2 + x\), then you can use the technique to write it as \(x(x + 1)\).

The technique of taking out common factors is useful not only for rearranging equations, but in many other situations where algebra is used. In this section you’ll learn about this technique and you’ll see how you can use the skill of rearranging equations to find the answers to some practical problems.

In Chapter 4 you’ll see that there’s a reverse process to multiplying out the brackets: sometimes an expression can be made simpler or more useful by *introducing* brackets.
2.9 Finding common factors

The way to reverse the process of multiplying out brackets is to consider the factors of the terms in the expression. A factor of a term is similar to a factor of an integer, which was discussed in Chapter 1.

Remember that a positive integer that divides a given integer exactly is called a factor of that integer. For example, 4 and 5 are both factors of 20 because

\[ 20 = 4 \times 5. \]

A similar idea applies to terms. If a term can be written in the form

\[ \text{something} \times \text{something}, \]

then both ‘somethings’ are factors of the term.

For example, consider the term \( a^2b \). We can write

\[ a^2b = a \times ab, \]

so \( a \) and \( ab \) are factors of \( a^2b \). Similarly,

\[ a^2b = a^2 \times b \quad \text{and} \quad a^2b = 1 \times a^2b, \]

so \( a^2 \), \( b \), 1 and \( a^2b \) are also factors of \( a^2b \).

Example 19 Checking a factor

Show that \( 3xy \) is a factor of \( 3xy^3 \), by writing \( 3xy^3 \) in the form

\[ 3xy \times \text{something}. \]

Solution

\[ 3xy^3 = 3xy \times y^2 \]

Activity 25 Checking factors

(a) Write \( pqr \) in the form \( q \times \text{something} \).
(b) Write \( A^7 \) in the form \( A^4 \times \text{something} \).
(c) Write \( 4f^3 \) in the form \( 2f \times \text{something} \).
(d) Write \( p^3q^5 \) in the form \( p^2q^2 \times \text{something} \).
(e) Write \( 6x^5y^8 \) in the form \( 2x^2y \times \text{something} \).

Remember also from Chapter 1 that if a positive integer is a factor of each of several integers, then it is a common factor of those integers. For example, 2 is a common factor of 8, 12 and 20, because

\[ 8 = 2 \times 4, \quad 12 = 2 \times 6 \quad \text{and} \quad 20 = 2 \times 10. \]

Again, a similar idea applies to terms. If something is a factor of each of several terms, then it is a common factor of those terms. For example, \( a \) is a common factor of

\[ a^2b \quad \text{and} \quad abc, \]

because

\[ a^2b = a \times ab \quad \text{and} \quad abc = a \times bc. \]
Example 20  Checking a common factor

Show that \( pq \) is a common factor of \( pq^2 \), \( 3p^2q^2 \) and \( pq \).

Solution

Write each term in the form \( pq \times \) something.

\[
\begin{align*}
pq^2 &= pq \times q, \\
3p^2q^2 &= pq \times 3pq \\
pq &= pq \times 1.
\end{align*}
\]

So \( pq \) is a common factor of the three terms.

Activity 26  Checking common factors

(a) Show that \( z \) is a common factor of \( 2z \) and \( z^2 \).
(b) Show that \( p^2 \) is a common factor of \( p^2q^2 \) and \( p^2 \).
(c) Show that \( 2AB \) is a common factor of \( 2A^2B^3 \), \( 4A^2B \) and \( 8AB \).

In Chapter 1 you saw that a collection of integers can have several common factors, and the largest of them is called the highest common factor. For example, the common factors of 8, 12 and 20 are 1, 2 and 4, and the highest common factor is 4.

Again, this idea also applies to terms: you can find the highest common factor of several terms. For example, consider again the terms \( a^2b \) and \( abc \).

You have seen that one common factor of these terms is \( a \), because

\[
\begin{align*}
a^2b &= a \times ab \\
abc &= a \times bc.
\end{align*}
\]

Another common factor is \( ab \), because

\[
\begin{align*}
a^2b &= ab \times a \\
abc &= ab \times c.
\end{align*}
\]

The common factor \( ab \) is a higher common factor than \( a \), as it is \( a \) multiplied by another factor. In fact, \( ab \) is the highest common factor of the two terms, since you cannot multiply \( ab \) by any other letters or integers (except 1) and still get a common factor of the two terms.

The next example shows you how to find the highest common factor of two terms.

Example 21  Finding a highest common factor

Find the highest common factor of the terms

\[
6ab^2c^2 \quad \text{and} \quad 9a^2b^5,
\]

and write each term in the form

\[
\text{highest common factor} \times \text{something}.
\]

Solution

First, consider the coefficients. The largest integer that divides both 6 and 9 exactly is 3.
Next, consider the powers of $a$. The largest power of $a$ that divides both $a$ and $a^2$ exactly is $a$.

Then consider the powers of $b$. The largest power of $b$ that divides both $b^7$ and $b^5$ exactly is $b^5$.

Finally, consider the powers of $c$. The second term doesn’t contain $c$ at all.

So, the highest common factor of the two terms is $3ab^5$.

The terms can be written as

$$6ab^7c^2 = 3ab^5 \times 2b^2c^2 \quad \text{and} \quad 9a^2b^5 = 3ab^5 \times 3a.$$ 

---

**Activity 27  Finding highest common factors**

In each of the following parts, find the highest common factor of the terms and write each term in the form

highest common factor $\times$ something.

(a) $2ab^2$ and $4ab$
(b) $3xy$ and $6y$
(c) $4p^3$, $9p^2$ and $2p^5$
(d) $10r$ and $15s$

---

**2.10 Taking out common factors**

Consider the expression

$$c^2d + cef.$$ 

The two terms in this expression, $c^2d$ and $ce$, have $c$ as a common factor.

So the expression can be written as

$$c \times cd + c \times ef.$$ 

From your work on multiplying out brackets, you know that this is the same as

$$c(cd + ef).$$ 

You can now see how to reverse the process of multiplying out brackets. First, you find a common factor of the terms, and write each term in the form

common factor $\times$ something.

Then you write the common factor at the front of a pair of brackets, and inside the brackets you write what is left of each term (the ‘something’). This is called taking out a common factor, or **factorising** the expression.
**Example 22  Taking out a common factor**

Factorise the expression $3r s^3 + r s$.

**Solution**

A common factor of the terms is $r s$.

$$3r s^3 + r s = r s \times 3s^2 + r s \times 1 = r s(3s^2 + 1)$$

You can take out any common factor of the terms in an expression, but it is usually best to take out the highest common factor. Do this in the next activity. Remember that you can always check a factorisation by multiplying out the brackets and checking that the expression that you get is the same as the one you factorised.

**Activity 28  Taking out common factors**

Factorise the following expressions.

(a) $ab + a^2$  
(b) $x^3y + yz$  
(c) $2w^2 + w^3$  
(d) $2z + 6z^4$

Expressions containing minus signs can be factorised in the same way.

**Example 23  Factorising an expression containing minus signs**

Factorise the expression $3m^3 - 6m^2 + 3m^4$.

**Solution**

The highest common factor of the terms is $3m^2$.

$$3m^3 - 6m^2 + 3m^4 = 3m^2 \times m - 3m^2 \times 2 + 3m^2 \times m^2 = 3m^2(m - 2 + m^2)$$

**Activity 29  Factorising expressions containing minus signs**

Factorise the following expressions.

(a) $2ab + 2b - 6b^2$  
(b) $A^5 - A^4$
As you get used to taking out common factors, you’ll probably find that you can skip the step of writing each term in the form

common factor \times something,

and just follow the shorter strategy below.

**Strategy**  To take out a common factor from an expression

1. Find a common factor of the terms (normally the highest common factor).
2. Write the common factor in front of a pair of brackets.
3. Write what’s left of each term inside the brackets.

---

**Example 24  Factorising efficiently**

Factorise the expression $-8X^2 + 2X + 2XY$.

**Solution**

The highest common factor is $2X$.

$-8X^2 + 2X + 2XY = 2X(-4X + 1 + Y)$

When you use the shorter strategy above, remember that if the common factor that you’re taking out is the same as one of the terms, then ‘what’s left’ of the term is 1. In Example 24, taking the factor $2X$ out of the term $2X$ left 1.

Try the shorter strategy in the following activity.

---

**Activity 30  Factorising efficiently**

Factorise the following expressions.

(a) $ab - 9bc$  
(b) $x^2 - x^5 + 2x^3$  
(c) $-2rs + 4r^2s^2$  
(d) $x\sqrt{y} - \sqrt{y}$

Once you have factorised an expression, you should check that you have taken out the *highest* common factor. To do this, check whether the terms inside the brackets have a common factor. If they do, take it out as well.

For example, suppose that you have carried out the following factorisation:

$de^2 - d^2e^2 + de^3 = de(e - de + e^2)$.

The terms inside the brackets have the common factor $e$, so you did not take out the highest common factor. Your working would continue as follows.

$= de \times e(1 - d + e)$

$= de^2(1 - d + e)$.

Now the terms inside the brackets have nothing in common, so $de^2$ is the highest common factor that can be taken out.
Many expressions cannot be factorised at all. For example, the terms in the expression
\[2de + 3ef + 4fd\]
have no common factor that can be usefully taken out.

### Activity 31  Taking out more common factors

Factorise the following expressions, where possible.

(a) \[12u + 6u^3 - 9u^2\]  
(b) \[5r^2 - 10\]  
(c) \[3fg - 2gh + 6fh\]  
(d) \[-8ABC - 4AB^2 + 2AB\]

If the coefficients of the terms of an expression are not integers, then you can often still factorise the expression.

### Example 25  Working with non-integer coefficients

Factorise the following expressions.

(a) \[0.2a - 0.8a^2\]  
(b) \[\frac{1}{x} - \frac{3}{7}q\]

**Solution**

(a) \[0.2a - 0.8a^2 = 0.2a(1 - 4a)\]  
(b) \[\frac{1}{x} - \frac{3}{7}q = \frac{1}{x}(1 - 3q)\]

### Activity 32  Working with non-integer coefficients

Factorise the following expressions.

(a) \[0.3m^2 - 0.6m + 0.9\]  
(b) \[\frac{1}{2}x - \frac{1}{2}x^2\]

Earlier in the course you saw how to multiply out brackets with a minus sign in front. You just change the sign of each term in the brackets. For example,

\[-(a + 2b - 2c - d) = -a - 2b + 2c + d.\]  \(6\)

Sometimes it’s useful to carry out the reverse of this process; to start with an expression without brackets and rewrite it so that it has a minus sign in front of brackets.

For example, you could start with the expression on the right-hand side of equation (6) and rewrite it as the expression on the left-hand side. As you can see, to do this you just have to change the sign of each term in the brackets. In general, remember the following.

**To take a minus sign outside brackets**

Change the sign of each term in the brackets.
For example,
\[-1 - x - x^2 + x^3 = -(1 + x + x^2 - x^3)\].

If you want to factorise an expression, and all or most of the terms have minus signs, then it’s usually best to take out a minus sign as well as any other common factors. In the next example first a minus sign is taken out, and then a common factor. As you get more used to taking out common factors, you should find that you can do both these things in one step.

**Example 26  Taking out a minus sign**

Factorise the expression 
\[-a - ab + a^2 - a^3\].

**Solution**

First take out a minus sign, then take out the common factor \(a\).

\[-a - ab + a^2 - a^3 = -(a + ab - a^2 + a^3)\]

\[= -a(1 + b - a + a^2)\]

**Activity 33  Taking out minus signs**

For each of the following expressions, take out a minus sign and factorise the expression if possible.

(a) \(-2u^2 - 2u^3 - 4u^4\)  
(b) \(-1 - a + a^2\)  
(c) \(pq - p^2q - q^2p - p^2q^2\)

When you’re factorising expressions, remember that you can always check your answer. Just multiply out the brackets again and check that you get the original expression. A check is particularly useful when you’ve taken out a minus sign.
Solutions and comments on Activities

Activity 1
(a) These are unlike terms: the first is a term in \( b \), and the second is a term in \( b^2 \).
(b) These are like terms: both are terms in \( D \).
(c) These are like terms: both are terms in \( z \). (The first term has coefficient 1, and the second has coefficient \(-1\).)
(d) These are unlike terms: the first is a constant term, and the second is a term in \( m \).

Activity 2
(a) \( 8A + 7A = (8 + 7)A = 15A \)
(b) \(-5d + 8d - 2d = (-5 + 8 - 2)d = 1d = d \)
(1d is usually written as \( d \).)
(c) \(-7z + z = -\frac{7}{1}z + \frac{1}{1}z = (-7 + 1)z = -6z \)
(d) \(1.4pq + 0.7pq - pq = 1.4pq + 0.7pq - 1pq = (1.4 + 0.7 - 1)pq = 1.1pq \)
(e) \( \frac{1}{3}n^2 - \frac{1}{5}n^2 = \frac{3}{15}n^2 - \frac{1}{5}n^2 = \frac{3 - 1}{5}n^2 = \frac{2}{5}n^2 \)
(You should give the exact answer, \( \frac{2}{5}n^2 \), not an approximation such as \( 0.167n^2 \).)

Activity 3
(a) These are like terms: both are terms in \( ab \).
(b) These are like terms: both are terms in \( rst \).
(c) These are like terms: both are terms in \( xy \).
(The second term can be written as \(-3xy\).)
(d) These are like terms: both are terms in \( ac^2 \).
(The first term can be written as \( 4ac^2 \).
(e) These are like terms: both are terms in \( abc \).
(The second term can be written as \( abc \).)
(f) These are unlike terms. If we write the second term with the letters in alphabetical order, then it’s \( 9cd^2 \). So the first term is a term in \( c^2d \) (that is, \( c \times c \times d \)), and the second is a term in \( cd^2 \) (that is, \( c \times d \times d \)).
(g) These are unlike terms: the first is a term in \( A^2 \), and the second is a term in \( a^2 \).
(h) These are unlike terms: the first is a term in \( fh \), and the second is a term in \( gh \).
(i) These are like terms, as they’re both constant terms.

Activity 4
(a) \( 4A - 3B + 3C + 5A + 2B - A \)
\( = 4A + 5A - A - 3B + 2B + 3C \)
\( = 8A - B + 3C \)
(b) \(-8v + 7 - 5w - 2v - 8 \)
\( = -8v - 2v - 5w + 7 - 8 \)
\( = -10v - 5w - 1 \)
(c) \( 20y^2 + 10xy - 10y^2 - 5y - 5xy \)
\( = 20y^2 - 10y^2 + 10xy - 5xy - 5y \)
\( = 10y^2 + 5xy - 5y \)
(d) \(-4ef + 8e^2f + 10fe - 3f^2e \)
\( = -4ef + 8e^2f + 10ef - 3ef^2 \)
\( = -4ef + 10ef + 8e^2f - 3ef^2 \)
\( = 6ef + 8e^2f - 3ef^2 \)
(e) \( \frac{2}{3}a + \frac{1}{3}b + 2a + \frac{1}{3}b \)
\( = \frac{2}{3}a + 2a + \frac{1}{3}b + \frac{1}{3}b \)
\( = \frac{2}{3}a + \frac{2}{3}a + \frac{1}{3}b + \frac{1}{3}b \)
\( = \frac{4}{3}a + \frac{2}{3}b \)

Activity 5
(a) \( 2a^3 - 3a - 2a^3 - 3a = -6a \)
(b) \( 2m + n - 5m + 2n + 3m = 3n \)
(c) \( b + 2b + 3b - 6b = 0 \)

Activity 6
(a) The formula is \( A = 10c + 2a \).
(b) The formula is \( T = 7c + 14a \).
(c) The formula is \( C = 10c + 2a + 7c + 14a \).
(d) Collecting like terms gives \( C = 17c + 16a \).
(e) Substituting \( c = 22 \) and \( a = 10 \) in the formula found in part (d) gives \( C = 17 \times 22 + 16 \times 10 = 374 + 160 = 534 \).
The cost of the trip is \( £534 \).
Activity 7
(a) \( y \times z \times 6 \times x \times 4 = 24xyz \)
(b) \( 7p \times 2qr = 14pqr \)
(c) \( QR \times G \times 5P = 5FGQR \)
(d) \( 2 \times a \times a \times 3 \times a = 6a^3 \)
(e) \( m \times n \times m \times 4 = 4m^2n \)
(f) \( 5y \times 2yx = 10xy^2 \)
(g) \( 4AB \times 4AB = 16A^2B^2 \)

Activity 8
(a) \( 8p^8 \times 5P = 40P^9 \)
(b) \( 2c^{10}d^3 \times 2c^2d^3 = 4c^{12}d^6 \)

Activity 9
(a) \( 9X \times (-XY) = -9X^2Y \)
(b) \( 3s \times \frac{1}{r} = 1rs = rs \)
(c) \(-3a^3 \times (-4a^4) = 12a^7 = 12a^7 \)
(d) \(-2pq \times (-3pq^2) = 6p^3q^2 = 6p^3q^2 \)
(e) \(-0.5g \times 2f^5 = -1f^5g = -f^5g \)
(f) \(-a \times b \times (-c) \times (-d) = -abed \)
(g) \((-x) \times (-y) \times (-x^2) \times (-4y) \)
\[ = +4x^3y^2 = 4x^3y^2 \]
(h) \((-3cd)^2 = (-3cd) \times (-3cd) = +9c^2d^2 = 9c^2d^2 \)
(i) \(-3cd)^2 = (-3cd \times 3cd) = -9c^2d^2 \)
You could do parts (c), (d), (g) and (h) in one step if you prefer.

Activity 10
(a) \ (+(-ab) = -ab \)
(b) \(-(-6x^2) = +6x^2 = 6x^2 \)
(c) \(-2M^4 = -2M^4 \)
(d) \(+(-7y) = -7y \)
(e) \(+5p = +5p = 5p \)
(f) \(-(-3n) = \frac{3}{4}n \)

Activity 11
(a) \( \frac{5m \times 2m}{2n \times n^2} = 10m^2 - 2n^3 \)
(b) \( 3p \times 2q + 2r \times (-7p) = 6pq - 14pr \)
(c) \( 2P - (-3Q) + (-P) + (2Q) = 2P + 3Q - P + 2Q \)
\[ = P + 5Q \]
(d) \(3 \times (-2a) - 1c^2 + 9ac = -6a - c^2 + 9ac \)
(Only the first and second terms were simplified. The third term was already in its simplest form.)
(e) \( 4s \times \frac{1}{2rst} = 2(-\frac{1}{2}s) = 2rs^2t + s \)
(f) \(-5xy + (-3y \times x^2) - (-y^2) = -5xy - 3x^2y + y^2 \)
(Only the second and third terms were simplified. The first term was already in its simplest form.)
(g) \(-3r \times (-2r) - (-2r \times r) + (r^2 \times 9) = 6r^2 + 2r^2 + 9r^2 = 17r^2 \)

Activity 12
(a) \( (a + b) \div 3 = \frac{a + b}{3} \)
(b) \( a + b \div 3 = \frac{a + b}{3} \)
(c) \( (x + 2) \div (y + 3) = \frac{x + 2}{y + 3} \)
(d) \( (x + 2) \div y + 3 = \frac{x + 2}{y} + 3 \)
(e) \( x + 2 \div (y + 3) = x + \frac{2}{y + 3} \)
(f) \( x + 2 \div y + 3 = x + \frac{2}{y} + 3 \)

Activity 13
(a) \( 6 \left( \frac{1 + \frac{h}{2}}{2} \right) = 6 + \frac{6h}{2} = 6 + 3h \)
(b) \( 6 \left( \frac{1 + \frac{h}{2}}{2} \right) = \frac{6}{2}(1 + h) = 3(1 + h) = 3 + 3h \)

Activity 14
(a) \( \frac{A - 6B}{3} = \frac{A}{3} - \frac{6B}{3} = \frac{A}{3} - 2B \)
(b) \( \frac{10z^2 + 5z - 20}{5} = \frac{10z^2}{5} + \frac{5z}{5} - \frac{20}{5} \)
\[ = 2z^2 + z - 4 \]
(c) \( \frac{3A^2 + A}{A} = \frac{3A^2}{A} + \frac{A}{A} = 3A + 1 \)

Activity 15
(a) \( \frac{x^4}{x^3} = \frac{\frac{1}{x^4}}{\frac{1}{x^3}} = \frac{1}{x} \)
(b) \( \frac{20a^2b}{15ab^2} = \frac{20a^2b}{31b} \)
(c) \( \frac{x^2 + 6x}{3x^2} = \frac{(x + 6)}{3x^2} = \frac{1}{3x} \)
(d) \[ \frac{u^2 - 4}{u^2 + 4u + 4} = \frac{(u - 2)(u + 2)}{(u + 2)^2} = \frac{1}{u + 2} = \frac{u - 2}{u + 2} \]

**Activity 16**

(a) \[ \frac{6}{y} + \frac{1}{y} = \frac{6 + 1}{y} = \frac{7}{y} \]

(b) \[ \frac{x}{a^2} + \frac{y}{a^2} = \frac{z}{a^2} \]

(c) Use 6xy as a common denominator:
\[ \frac{a}{2x} - \frac{b}{3y} = \frac{3ay}{6xy} - \frac{2bx}{6xy} = \frac{3ay - 2bx}{6xy} \]

(d) Use 3x(x + 3) as a common denominator:
\[ \frac{1}{3x} - \frac{2}{x + 3} = \frac{3x(x + 3) - 6x}{3x(x + 3)} = \frac{3x + 6x}{3x(x + 3)} = \frac{3x(x + 3)}{3x(x + 3)} = \frac{5x + 3}{3x(x + 3)} \]

**Activity 17**

(a) Use \( x^4 \) as a common denominator:
\[ \frac{5}{x^4} - \frac{4}{x^2} = \frac{5x^2 - 4x^2}{x^4} = \frac{5 - 4x^2}{x^4} \]

(b) Use \( x(x + 1) \) as a common denominator:
\[ \frac{1}{x^2 + x} - \frac{1}{x + 1} = \frac{1}{x(x + 1)} - \frac{x}{x(x + 1)} = \frac{x - 1}{x(x + 1)} \]

(c) Use \( b + 3 \) as a common denominator:
\[ \frac{a}{b + 3} + \frac{a}{b + 3} = \frac{a(x + b + 3) + a(x + b + 3)}{b + 3} = \frac{ab + 4a}{b + 3} = \frac{a(b + 4)}{b + 3} \]

**Activity 18**

(a) \[ \frac{p^2}{q^2} \times \frac{p}{q} = \frac{p^2}{q^2} \times \frac{p}{q} = \frac{p^3}{q^3} \]

(b) \[ \frac{p^2}{q^2} \div \frac{p}{q} = \frac{p^2}{q^2} \times \frac{q}{p} = \frac{p^2 q}{q^2 p} = \frac{p}{q} \]

(c) \[ \frac{9ax^2}{b} \times \frac{b^3}{4xy^2} = \frac{9ax^2 b^3}{b} \times \frac{4xy^2}{b} = \frac{9axb^2}{4y} \]

(d) \[ \frac{9ax^2}{b} \div \frac{b^3}{4xy^2} = \frac{9ax^2}{b} \times \frac{4xy^2}{b^3} = \frac{9ax^2 y^2}{b^3} \]

(e) \[ 8u^2 \times \frac{2}{u^2 + u} = \frac{8u^2}{u^2 + u} \times \frac{2}{u^2 + u} = \frac{16u^2}{u^2 + u} = \frac{16u^2}{u(u + 1)} = \frac{16u}{u + 1} \]

**Activity 19**

(a) By the rule for dividing by a fraction,
\[ F = \frac{GM}{R^2} \div \frac{Gm}{r^2} = \frac{GM}{R^2} \times \frac{r^2}{Gm} = \frac{Mm^2}{mR^2} \]

(b) We substitute \( M = 80m \) and \( R = 4r \) in the formula obtained in part (a):
\[ F = \frac{80Mr^2}{m(4r)^2} = \frac{80mr^2}{16mr^2} = 5, \]
so \( F = 5f \).

**Activity 20**

(a) \( 3p(qg + 4) = 3p \times qg + 3p \times 4 = 3p^2q + 12p \)

(b) \( ax^2(x + 2y) = ax^2 \times x + ax^2 \times 2y = ax^3 + 2ax^2y \)

**Activity 21**

(a) \( f(c + 5g) = ef + 5fg \)

(b) \( 5(2A + B) = 10A + 5B \)

(c) \( 3c(4c + 2d) = 12c^2 + 6cd \)

(d) \( (a - b)c^2 = ac^2 - bc^2 \)

(e) \( 2y(x + 2y + 4z) = 2xy + 4y^2 + 8yz \)

(f) \( 2 \left( \frac{1}{2}A^2 + \frac{2}{3} \right) = A^2 + 3 \)

(g) \( a(x + y)z = axz + ayz \)

(h) \( 2b(b^2 + 2b^3) = 2b^3 + 4b^5 \)

**Activity 22**

(a) \( p(q - r) = pq - pr \)

(b) \( 7a(-4a + 3b) = -28a^2 + 21ab \)

(c) \( 6(0.2a - 0.3b + 1.4) = 1.2a - 1.8b + 8.4 \)

(d) \( 10(\frac{1}{2}n + \frac{1}{3}) = 5n + 2 \)

(e) \( -3(x - 2y) = -3x + 6y \)

(f) \( -b^2(-a + b) = ab^2 - b^3 \)

**Activity 23**

(a) \( f + g(f + h) = f + fg + gh \)

(b) \( x - y(x + 2y) = x - xy - 2y^2 \)

(c) \( 2p - 3q(-3p + q) = 2p + 9pq - 3q^2 \)

(d) \( -2(a + b) + 4(a - b) = -2a - 2b + 4a - 4b = 2a - 6b \)

(e) \( 2aE - 3E(-E - 5a) = 2aE + 3E^2 + 15aE = 17aE + 3E^2 \)
Activity 24

(a) \(-4f - g^3\) = 
\[-4f + g^3\]
(b) \(-x + 7y - 8z + 6\) = 
\[x - 7y + 8z - 6\]
(c) \(2(a - b) + (c - 2d)\) = 
\[2a - 2b + c - 2d\]
(d) \(r + (−2s − r)\) = 
\[r - 2s - r = -2s\]
(e) \(-A + B - (−3A + 4B)\) = 
\[-A + B + 3A - 4B = 2A - 3B\]
(f) \(-(−t − w) + (−t + w)\) = 
\[t + w - t + w = 2w\]
(g) \(-(L + 2M) − (−M)\) = 
\[-L - 2M + M = -L - M\]

Activity 25

(a) \(pqr = q \times pr\)
(b) \(A^7 = A^4 \times A^3\)
(c) \(4f^3 = 2f \times 2f^2\)
(d) \(p^3q^5 = p^2q^2 \times pq^3\)
(e) \(6x^5y^8 = 2x^2y \times 3x^3y^7\)

Activity 26

(a) \(2z = z \times 2\) and \(z^2 = z \times z\).
So \(z\) is a common factor of the two terms.
(b) \(p^2q^2 = p^2 \times q^2\) and \(p^2 = p^2 \times 1\).
So \(p^2\) is a common factor of the two terms.
(c) \(2A^2B^2 = 2AB \times AB, 4A^2B = 2AB \times 2A\) and 
\(8AB = 2AB \times 4\).
So \(2AB\) is a common factor of the three terms.

Activity 27

(a) The highest common factor of \(2ab^2\) and \(4ab\) is \(2ab\).
\(2ab^2 = 2ab \times b\) and \(4ab = 2ab \times 2\).
(b) The highest common factor of \(3xy\) and \(6y\) is \(3y\).
\(3xy = 3y \times x\) and \(6y = 3y \times 2\).
(c) The highest common factor of \(4p^3, 9p^2\) and 
\(2p^5\) is \(p^2\).
\(4p^3 = p^2 \times 4p, 9p^2 = p^2 \times 9\) and 
\(2p^5 = p^2 \times 2p^3\).
(d) The highest common factor of \(10r\) and \(15s\) is \(5\).
\(10r = 5 \times 2r\) and \(15s = 5 \times 3s\).

Activity 28

(a) \(ab + a^2 = a \times b + a \times a = a(b + a)\)
(b) \(x^3y + yz = y \times x^3 + y \times z = y(x^3 + z)\)
(c) \(2w^2 + w^3 = w^2 \times 2 + w^2 \times w = w^2(2 + w)\)
(d) \(2z + 6z^4 = 2z \times 1 + 2z \times 3z^3 = 2z(1 + 3z^3)\)

Activity 29

(a) \(2ab + 2b - 6b^2 = 2b \times a + 2b \times 1 - 2b \times 3b\) = 2b(a + 1 - 3b)
(b) \(A^5 - A^4 = A^4 \times A - A^4 \times 1 = A^4(A - 1)\)

Activity 30

(a) \(ab - 9bc = b(a - 9c)\)
(b) \(x^2 - x^5 + 2x^3 = x^2(1 - x^3 + 2x)\)
(c) \(-2rs + 4r^2s^2 = 2rs(-1 + 2rs)\)
(d) \(x\sqrt{y} - \sqrt{x} = \sqrt{y}(x - 1)\)

Activity 31

(a) \(12u + 6u^3 - 9u^2 = 3u(4 + 2u^2 - 3u)\)
(b) \(5r^2 - 10 = 5(r^2 - 2)\)
(c) The terms of the expression \(3fg - 2gh + 6fh\) have no common factors.
(d) \(-8ABC - 4AB^2 + 2AB = 2AB(-4C - 2B + 1)\)

Activity 32

(a) \(0.3m^2 - 0.6m + 0.9 = 0.3(m^2 - 2m + 3)\)
(b) \(\frac{1}{7}x - \frac{1}{3}x^2 = \frac{1}{7}x(1 - x)\)

Activity 33

(a) \(-2u^2 - 2u^3 - 4u^4 = -(2u^2 + 2u^3 + 4u^4)\) = \(-2u^2(1 + u + 2u^2)\)
(b) \(-1 - a + a^2 = -(1 + a - a^2)\)
(c) \(pq - p^2q - q^2p - p^2q^2\) = \(-pq + p^2q + q^2p + p^2q^2\)
= \(-pq(-1 + p + q + pq)\)
= \(-pq(p + q + pq - 1)\)
(In the final step of part (c), the order of the terms inside the brackets has been changed, so that the term with a minus sign is not first. This is not essential, but it makes the expression slightly shorter and tidier.)