Pract \textit{ice} Exponentials and logarithms – set 1

\textbf{Exercise 1.1}

(a) Use your calculator, where necessary, to find image values of the function \( f(x) = 5^x \) for \( x = -1, -0.75, -0.5, \ldots, 1 \).

(b) Use your results from part (a) to sketch the graph of the function \( f(x) = 5^x \) for \( x \) in \([-1, 1]\).

(c) Explain how, without performing any further calculations, you could use the result from part (b) to sketch the graph of the function \( g(x) = (\frac{1}{5})^x \).

\textbf{Exercise 1.2}

By considering the effect of an appropriate translation on the graph of the function \( f(x) = e^x \), sketch the graph of the function \( g(x) = e^{x+1} \).

\textbf{Exercise 1.3}

Fill in the gaps in the table below.

<table>
<thead>
<tr>
<th>Power form</th>
<th>Logarithmic form</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 81 = 3^4 )</td>
<td>( \log_2 4 = 2 )</td>
</tr>
<tr>
<td>( 1 = 6^0 )</td>
<td>( \log_5 125 = 3 )</td>
</tr>
<tr>
<td>( 0.2 = 5^{-1} )</td>
<td>( \log_5 8 = 2 )</td>
</tr>
<tr>
<td>( 5 = 5^1 )</td>
<td>( x = 2^{-4} )</td>
</tr>
</tbody>
</table>

\textbf{Exercise 1.4}

(a) Find the logarithm to the base 2 of each of the following.

(i) 16 (ii) 2 (iii) 0.5

(b) Without using a calculator, find the logarithm to the base 10 of each of the following.

(i) 1000 (ii) 10 (iii) 0.1

You will have noticed that \( \log_{10} 10 \) and \( \log_2 2 \) have the value 1. This result is generally true, i.e. \( \log_a a = 1 \).

\textbf{Exercise 1.5}

Simplify each of the following.

(a) (i) \( \log_{10} 12 + \log_{10} 3 \) (ii) \( \log_{10} 12 - \log_{10} 6 \) (iii) \( 3 \log_{10} 5 \)

(b) (i) \( \ln 15 + \ln 7 \) (ii) \( \ln 35 - \ln 14 \) (iii) \( 4 \ln 3 \)

\textbf{Exercise 1.6}

Solve each of the following exponential equations, giving your answer correct to three decimal places.

(a) \( 3^x = 5 \) (b) \( 5^x = 3 \) (c) \( 12^{-x} = 5 \) (d) \( 4^{x-1} = 4 \)
Solutions to Exercises

Solution 1.1

(a) The required values (correct to two decimal places) of $5^x$ are shown in the following table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-1$</th>
<th>$-0.75$</th>
<th>$-0.5$</th>
<th>$-0.25$</th>
<th>$0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5^x$</td>
<td>0.2</td>
<td>0.30</td>
<td>0.45</td>
<td>0.67</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5^x$</td>
<td>1</td>
<td>1.50</td>
<td>2.24</td>
<td>3.34</td>
<td>5</td>
</tr>
</tbody>
</table>

(b) The graph is shown in Figure S.1.

![Figure S.1](https://via.placeholder.com/150)

Figure S.1

(c) We have $g(x) = (\frac{1}{5})^x = 5^{-x}$. The graph of this function is obtained from that of $f(x) = 5^x$ by reflection in the $y$-axis, which is equivalent to an $x$-scaling with factor $-1$. Hence the graph of $g(x) = (\frac{1}{5})^x$ is as in Figure S.2.

![Figure S.2](https://via.placeholder.com/150)

Figure S.2

Solution 1.2

The graph of $g(x) = e^{x+1}$ is obtained from that of $f(x) = e^x$ by applying a horizontal translation by 1 unit to the left. This gives the graph in Figure S.3.

![Figure S.3](https://via.placeholder.com/150)

Figure S.3

(Note that the graph of $g$ may also be obtained from that of $f$ by applying a $y$-scaling with factor $e$, since $e^{x+1} = e \times e^x$.)

Solution 1.3

<table>
<thead>
<tr>
<th>Power form</th>
<th>Logarithmic form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$81 = 3^4$</td>
<td>$\log_{10} 81 = 4$</td>
</tr>
<tr>
<td>$4 = 2^2$</td>
<td>$\log_{10} 4 = 2$</td>
</tr>
<tr>
<td>$1 = 6^0$</td>
<td>$\log_{10} 1 = 0$</td>
</tr>
<tr>
<td>$125 = 5^3$</td>
<td>$\log_{10} 125 = 3$</td>
</tr>
<tr>
<td>$0.2 = 5^{-1}$</td>
<td>$\log_{10} 0.2 = -1$</td>
</tr>
<tr>
<td>$5 = 5^1$</td>
<td>$\log_{10} 5 = 1$</td>
</tr>
<tr>
<td>$8 = x^2$</td>
<td>$\log_{10} 8 = 2$</td>
</tr>
<tr>
<td>$x = 2^{-4}$</td>
<td>$\log_{10} x = -4$</td>
</tr>
</tbody>
</table>

Solution 1.4

(a) (i) $16 = 2^4$, so $\log_2 16 = 4$.
    (ii) $2 = 2^1$, so $\log_2 2 = 1$.
    (iii) $0.5 = \frac{1}{2} = 2^{-1}$, so $\log_2 0.5 = -1$.

(b) (i) $1000 = 10^3$, so $\log_{10} 1000 = 3$.
    (ii) $10 = 10^1$, so $\log_{10} 10 = 1$.
    (iii) $0.1 = 10^{-1}$, so $\log_{10} 0.1 = -1$.

Solution 1.5

(a) (i) $\log_{10} 36$ (ii) $\log_{10} 2$ (iii) $\log_{10} 125$
(b) (i) $\ln 105$ (ii) $\ln 2.5$ (iii) $\ln 81$
Solution 1.6

Brief solutions are given.

(a) $3^x = 5$
   
   $x \log 3 = \log 5$
   
   $x = \frac{\log 5}{\log 3}$
   
   $= 1.465$ (to 3 d.p.)

(b) $5^x = 3$
   
   $x = \frac{\log 3}{\log 5}$
   
   $= 0.683$ (to 3 d.p.)

(c) $12^{-x} = 5$
   
   $-x = \frac{\log 5}{\log 12}$
   
   $x = -\frac{\log 5}{\log 12}$
   
   $= -0.648$ (to 3 d.p.)

(d) $4^{x-1} = 4$
   
   $(x - 1) \log 4 = \log 4$
   
   $x - 1 = 1$
   
   $x = 2$