**Practice** Trigonometry – set 2

**Exercise 2.1**
Which trigonometric ratio would you use to find the length of the side marked $x$ in each of these triangles?

![Diagrams of triangles for Exercise 2.1](image)

**Exercise 2.2**
Where possible, find the length of each side marked $x$ in Exercise 2.1.
**Exercise 2.3**

Which trigonometric ratio would you use to find the angle marked $\theta$ in each of these triangles?

![Diagram](image)

**Exercise 2.4**

Find the size of each angle marked $\theta$ in Exercise 2.3.

**Exercise 2.5**

Philip plans to install photovoltaic cells on the roof of his house to generate electricity. The amount of electricity generated depends partly on the angle of elevation of the roof. To find this, Philip takes a photograph of the gable end of his house and draws this sketch from it.

![Diagram](image)

(a) Assuming that the triangle is symmetrical, find angle $\theta$ in the sketch.

(b) Do you think that this gives a good estimate of the angle of elevation of the roof?

**Exercise 2.6**

(a) Calculate the angle that the line $y = 2x$ makes with the positive $x$-axis. Now find the angle that the line $y = 3x$ makes with the positive $x$-axis.

(b) What is the relationship between $m$, the gradient of the straight line $y = mx$, and $\theta$, the angle that the line makes with the positive $x$-axis?
**Exercise 2.7**

Find the length of each side marked $x$ in the following triangles.

(a)

(b)

(c)

(d)

**Exercise 2.8**

Find the distance $d$ across the pond.
Exercise 2.9
Find the height of the hill, CD. Try to work out how to do it yourself first; there is a hint underneath the diagram if you need it.

(Hint: Use triangle ABD to find the length of AD. Then use triangle ACD to find CD.)

Exercise 2.10
Tim usually travels from A to B via M but a bridge is closed on the road between M and B. Which is now his shortest route? AM = 10 km and MB = 12 km.
Exercise 2.11
Find each angle marked $\theta$ in the following triangles.

Exercise 2.12
Use three different formulas to find the area of this triangle.

Exercise 2.13
(a) Match the angles.

<table>
<thead>
<tr>
<th>Degrees</th>
<th>0</th>
<th>90</th>
<th>45</th>
<th>360</th>
<th>180</th>
<th>60</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radians</td>
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<td>$\pi$</td>
<td>$\pi$</td>
<td>$\pi$</td>
<td>$\pi$</td>
<td>$\pi$</td>
<td>$\pi$</td>
</tr>
</tbody>
</table>

(b) Match the values.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>$\cos \frac{\pi}{6}$</th>
<th>$\tan 30^\circ$</th>
<th>$\sin \frac{\pi}{4}$</th>
<th>$\cos 60^\circ$</th>
<th>$\tan \frac{\pi}{3}$</th>
<th>$\sin 90^\circ$</th>
<th>$\cos \frac{\pi}{2}$</th>
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<tbody>
<tr>
<td>Exact value</td>
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<td>$\sqrt{3}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$1$</td>
<td>$1$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Exercise 2.14
(a) Convert $120^\circ$ into radians.
(b) Convert $\frac{\pi}{12}$ radians into degrees.
Solutions to Exercises

Solution 2.1
(a) Sine (b) Cosine (c) Cosine
(d) Tangent
(e) It is not necessary to use a trigonometric ratio because it is an isosceles triangle.
(f) This is not a right-angled triangle, and you would need more information (another angle, perhaps, to use the Sine Rule) to find x.

Solution 2.2
(a) \( \sin 30^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{x}{29} \)
So \( x = 29 \sin 30^\circ = 14.5 \).
(b) \( \cos 47^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{x}{2.5} \)
So \( x = 2.5 \cos 47^\circ = 1.704 \ldots \)
\( x = 1.70 \) (to 3 s.f.)
(c) \( \cos 58^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{13}{x} \)
So \( x = \frac{13}{\cos 58^\circ} = 24.532 \ldots \)
\( x = 24.5 \) (to 3 s.f.)
(d) \( \tan 34^\circ = \frac{\text{opp}}{\text{adj}} = \frac{4}{x} \)
So \( x = \frac{4}{\tan 34^\circ} = 5.930 \ldots \)
\( x = 5.93 \) (to 3 s.f.)
(e) This is an isosceles triangle so \( x = 7 \).
(f) There is not sufficient information given to calculate \( x \).

Solution 2.3
(a) Tangent (b) Cosine (c) Cosine
(d) Sine

Solution 2.4
(a) \( \tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{5} \)
So \( \theta = \tan^{-1} \left( \frac{4}{5} \right) = 38.659 \ldots ^\circ \)
\( \theta = 39^\circ \) (to the nearest degree)
(b) \( \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{24}{29} \)
So \( \theta = \cos^{-1} \left( \frac{24}{29} \right) = 34.148 \ldots ^\circ \)
\( \theta = 34^\circ \) (to the nearest degree)

Solution 2.5
(a) Divide the big triangle into two right-angled triangles.
(b) The triangle in Philip’s sketch should be a scaled version of the actual gable end, and so it would be reasonable to assume that the triangles are similar and that \( \theta \) would be a reasonable estimate of the angle of elevation of the roof. However, the sketch has been drawn from a photograph and therefore it is unlikely that the side lengths in the sketch are exactly in proportion with those in the gable end. So the angle calculated from these lengths can only be approximate.

(c) \( \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{13}{22} \)
So \( \theta = \cos^{-1} \left( \frac{13}{22} \right) = 53.778 \ldots ^\circ \)
\( \theta = 54^\circ \) (to the nearest degree)
(d) \( \sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{3.1}{3.5} \)
So \( \theta = \sin^{-1} \left( \frac{3.1}{3.5} \right) = 62.339 \ldots ^\circ \)
\( \theta = 62^\circ \) (to the nearest degree)
**Solution 2.6**

(a) Let the angle that the line \( y = 2x \) makes with the positive \( x \)-axis be \( \theta \). In the triangle, the line opposite angle \( \theta \) is 2 units and the side adjacent to angle \( \theta \) is 1 unit.

\[
\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{2}{1} = 2
\]

So \( \theta = \tan^{-1}(2) = 63.4349\ldots^\circ \)

So the angle that the line \( y = 2x \) makes with the positive \( x \)-axis is 63\(^\circ\) (to the nearest degree).

Similarly the angle that the line \( y = 3x \) makes with the positive \( x \)-axis is

\[
\tan^{-1}(3) = 71.565\ldots^\circ = 72^\circ \text{ (to the nearest degree)}
\]

(b) The angle between the line \( y = mx \) and the positive \( x \)-axis is \( \tan^{-1}(m) \) so the gradient of a straight line is also the tangent of the angle between the line and the positive \( x \)-axis.

**Solution 2.7**

(a) Using the Sine Rule,

\[
\frac{x}{\sin 65^\circ} = \frac{13}{\sin 40^\circ}
\]

So \( x = \frac{13\sin 65^\circ}{\sin 40^\circ} = 18.329\ldots \)

\( x = 18.3 \) (to 3 s.f.)

(b) Angle \( F = 180^\circ - 80^\circ - 35^\circ = 65^\circ \).

Using the Sine Rule,

\[
\frac{x}{\sin 65^\circ} = \frac{7.2}{\sin 35^\circ}
\]

So \( x = \frac{7.2\sin 65^\circ}{\sin 35^\circ} = 11.376\ldots \)

\( x = 11.4 \) (to 3 s.f.)

(c) Using the Cosine Rule,

\[
x^2 = 11^2 + 6^2 - 2 \times 11 \times 6 \times \cos 72^\circ
\]

\[
= 116.209\ldots 
\]

\( x = 10.780\ldots = 10.8 \) (to 3 s.f.)

(d) Using the Cosine Rule,

\[
x^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 135^\circ
\]

\[
= 247.994\ldots 
\]

\( x = 15.747\ldots = 15.7 \) (to 3 s.f.)

**Solution 2.8**

Using the Cosine Rule,

\[
d^2 = 36^2 + 45^2 - 2 \times 36 \times 45 \times \cos 70^\circ
\]

\[
= 2212.854\ldots 
\]

\( d = 47.040\ldots \)

The distance across the pond is 47.0 m (to 1 d.p.)

**Solution 2.9**

Angle \( ABD = 180^\circ - 30^\circ = 150^\circ \) (angles on a straight line).

So angle \( ADB = 180^\circ - 150^\circ - 23^\circ = 7^\circ \) (angles in a triangle).

Using the Sine Rule in triangle \( ABD \),

\[
\frac{AD}{\sin 150^\circ} = \frac{56}{\sin 7^\circ}
\]

So \( AD = \frac{56 \sin 150^\circ}{\sin 7^\circ} \)

In triangle \( ACD \),

\[
\sin 23^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{CD}{AD}
\]

So \( CD = \frac{56 \sin 150^\circ}{\sin 7^\circ} \times \sin 23^\circ = 89.772\ldots \text{m} \)

The hill is 89.8 m high (to 1 d.p.).

**Solution 2.10**

It is a shorter distance from \( A \) to \( C \) directly than going via \( M \) in either case. So there are two possible routes, \( ACB \) and \( ADB \).

In triangle \( AMC \),

\[
\cos 30^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{10}{AC}
\]

So \( AC = \frac{10 \cos 30^\circ}{10} = 11.547\ldots \text{km} \)

\[
\tan 30^\circ = \frac{\text{opp}}{\text{adj}} = \frac{CM}{10}
\]

So \( CM = 10 \tan 30^\circ = 5.773\ldots \text{km} \)

In triangle \( MCB \), angle \( CMB = 360^\circ - 90^\circ - 140^\circ - 30^\circ = 100^\circ \) (angles at a point). Using the Cosine Rule in triangle \( MCB \),

\[
CB^2 = (5.733\ldots)^2 + 12^2 - 2 \times 5.733 \times 12 \times \cos 100^\circ
\]

\[
= 201.394\ldots 
\]

\( CB = 14.191\ldots \text{km} \)

So the distance from \( A \) to \( B \) via \( C \) is

11.547\ldots + 14.191\ldots = 25.738\ldots = 25.7 \text{ km (to 1 d.p.)} \)
The third angle in triangle MAD is $180^\circ - 140^\circ - 20^\circ = 20^\circ$. So triangle MAD is isosceles. So $MD = 10$ km.

Using the Sine Rule in triangle MAD (though you can divide the triangle into two right-angled triangles if you prefer),

$$\frac{AD}{\sin 140^\circ} = \frac{10}{\sin 20^\circ}$$

So $AD = \frac{10 \sin 140^\circ}{\sin 20^\circ} = 18.793 \ldots$ km

Using the Cosine Rule in triangle DMB,

$$DB^2 = 10^2 + 12^2 - 2 \times 10 \times 12 \times \cos 30^\circ$$

$$DB = 6.012 \ldots$$ km

So the distance from A to B via D is $18.793 \ldots + 6.012 \ldots = 24.806 \ldots = 24.8$ km (to 1 d.p.)

So the shortest route is $ADB$.

**Solution 2.11**

(a) The angle $\theta$ must be acute because it is opposite a short side of the triangle. Using the Sine Rule,

$$\frac{\sin \theta}{13} = \frac{\sin 65^\circ}{17}$$

$$\sin \theta = \frac{13 \sin 65^\circ}{17}$$

$$\theta = \sin^{-1} \left( \frac{13 \sin 65^\circ}{17} \right)$$

$$\theta = 43.872 \ldots ^\circ = 44^\circ \text{ (to the nearest degree)}$$

(b) Using the Cosine Rule,

$$7.2^2 = 10.4^2 + 11.5^2 - 2 \times 10.4 \times 11.5 \times \cos \theta$$

$$\cos \theta = \frac{10.4^2 + 11.5^2 - 7.2^2}{2 \times 10.4 \times 11.5} = 0.7883 \ldots$$

$$\theta = 37.96 \ldots ^\circ = 38^\circ \text{ (to the nearest degree)}$$

(c) Using the Cosine Rule,

$$15^2 = 6^2 + 12^2 - 2 \times 6 \times 12 \times \cos \theta$$

$$\cos \theta = \frac{6^2 + 12^2 - 15^2}{2 \times 6 \times 12} = -0.3125$$

$$\theta = 108.209 \ldots ^\circ = 108^\circ \text{ (to the nearest degree)}$$

(d) The angle $\theta$ could be either acute or obtuse here because it is opposite a long side of the triangle. Using the Sine Rule,

$$\frac{\sin \theta}{12} = \frac{\sin 54^\circ}{10}$$

$$\sin \theta = \frac{12 \sin 54^\circ}{10}$$

$$\theta = \sin^{-1} \left( \frac{12 \sin 54^\circ}{10} \right)$$

$$\theta = 76.124 \ldots ^\circ \text{ or } 180^\circ - 76.124 \ldots ^\circ$$

$$\theta = 103.875 \ldots ^\circ$$

So angle $\theta$ could be either $76^\circ$ or $104^\circ$ (to the nearest degree).

**Solution 2.12**

Using the formula, area = $\frac{1}{2} \times$ base $\times$ height, with $AB$ as the base, the area = $\frac{1}{2} \times 4 \times 3 = 6$ cm².

Using the formula, area = $\frac{1}{2} ab \sin \theta$ with angle $BAC$ as $\theta$ so that $\sin \theta = \frac{3}{5}$, the area = $\frac{1}{2} \times 4 \times 5 \times \left( \frac{3}{5} \right) = 6$ cm².

Using Heron's formula, area = $\sqrt{s(s - a)(s - b)(s - c)}$

where $s = \frac{1}{2}(a + b + c)$.

Then $s = \frac{1}{2}(3 + 4 + 5) = 6$ cm, and the area = $\sqrt{6 \times 3 \times 1 \times 2} = \sqrt{36} = 6$ cm².

**Solution 2.13**

(a)

Degrees 0 90 45 360 180 60 30

Radians 0 $\frac{\pi}{2}$ $\frac{\pi}{4}$ $2\pi$ $\pi$ $\frac{\pi}{3}$ $\frac{\pi}{6}$

(b)

Ratio $\cos \frac{\pi}{6}$ $\tan 30^\circ$ $\sin \frac{\pi}{4}$ $\cos 60^\circ$ $\tan \frac{\pi}{3}$ $\sin 90^\circ$ $\cos \frac{\pi}{2}$

Exact $\frac{\sqrt{3}}{2}$ $\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{2}}$ $\frac{1}{2}$ $\sqrt{3}$ 1 0

**Solution 2.14**

(a) Angle in radians = $\frac{\pi}{180} \times 120 = \frac{2\pi}{3}$

(b) Angle in degrees = $\frac{180}{\pi} \times \frac{\pi}{12} = 15$